Given: $A=\left(2\sqrt{3}-\sqrt{2}\right)^2+\sqrt{2}\left(\sqrt{2}+4\sqrt{3}\right)$; $B=\dfrac{\sqrt{12}}{\sqrt{12}-\sqrt{3}}$ and $C=\dfrac{(0.08)^2\times\sqrt{10^{-2}}\times\left(10^2\right)^4}{8000}$

1) Show that A is an integer.
2) Simplify B and C.
3) Deduce that $\frac{A}{B}=C$.

1) $A=\left(2\sqrt{3}-\sqrt{2}\right)^2+\sqrt{2}\left(\sqrt{2}+4\sqrt{3}\right)=12\text{-}4\sqrt{6}+2+2+4\sqrt{6}=16.$

2) $B=\dfrac{\sqrt{12}}{\sqrt{12}-\sqrt{3}}=\dfrac{2\sqrt{3}}{2\sqrt{3}-\sqrt{3}}=\dfrac{2\sqrt{3}}{\sqrt{3}}=2.$

$C=\dfrac{(0.08)^2\times\sqrt{10^{-2}}\times\left(10^2\right)^4}{8000}=\dfrac{64\times10^{-4}\times10^{-1}\times10^8}{8\times10^3}=\dfrac{64\times10^3}{8\times10^3}=8.$

3) $\dfrac{A}{B}=\dfrac{16}{2}=8$, then $\dfrac{A}{B}=C=8.$

Part A

Consider the expression: $E(x) = 2(x-2)(x+4)$.

1) Verify that $E(x) = 2x^2+4x-16$.

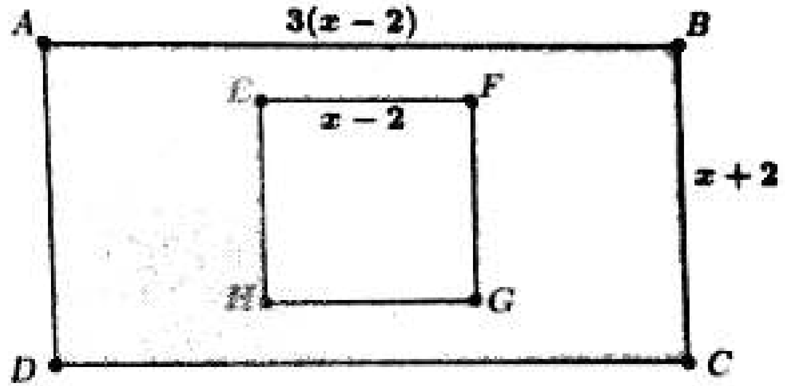
2) Solve the equation: $2x^2+4x-16=0$.

Part B

In the adjacent figure; the unit of length is the centimeter (x is an integer strictly greater than 2).

ABCD is a rectangle of area $S_1(x)$, such that $AB = 3(x-2)$ and $BC = (x+2)$

EFGH is a square of area $S_2(x)$, such that $EF = (x-2)$.



1) Express $S_1(x)$ and $S_2(x)$ in terms of x.

2) Prove that the area of the colored part is equal to: $2(x-2)(x+4)$.

3) Calculate x so that the area of ABCD is equal to four times the area of EFGH.

Part C

Let $R(x)$ be the ratio of the area of ABCD to that of EFGH.

1) Determine the values of x for which $R(x)$ is defined.

2) Simplify $R(x)$, then rationalize the denominator of $R(\sqrt{5})$.

3) Does the equation $R(x) = 3$ admit a solution? Justify.

A) 1) $E(x) = 2(x^2+4x-2x-8) = 2x^2+8x-4x-16 = 2x^2+4x-16$.

2) $E(x) = 0$ then $x-2 = 0$ or $x+4 = 0$ so $x = 2$ or $x = -4$.

B) 1) $S_1(x) = 3(x-2)(x+2)$; $S_2(x) = (x-2)^2$.

2) Area(shaded part) = $S_1(x)-S_2(x) = 3(x-2)(x+2)-(x-2)^2 = (x-2)[3x+6-x+2] = 2(x-2)(x+4)$.

3) $S_1(x) = 4S_2(x)$ then $3(x-2)(x+2) = 4(x-2)^2$ then $3(x-2)(x+2)-4(x-2)^2 = 0$

so $(x-2)[3x+6-4x+8] = 0$

then $(x-2)(-x+14) = 0$ then $x = 2$ (rejected) or $x = 14$ (accepted).

C) $R(x) = \frac{S_1(x)}{S_2(x)} = \frac{3(x-2)(x+2)}{(x-2)^2}$.

1) $(x-2)^2 \neq 0$, $x-2 \neq 0$, $x \neq 2$.

2) $R(x) = \frac{3(x-2)(x+2)}{(x-2)^2} = \frac{3(x+2)}{x-2}$.

$R(\sqrt{5}) = \frac{(3\sqrt{5}+6)(\sqrt{5}+2)}{(\sqrt{5}-2)(\sqrt{5}+2)} = \frac{15+6\sqrt{5}+6\sqrt{5}+12}{\sqrt{5}^2-2^2} = \frac{27+12\sqrt{5}}{1} = 27 + 12\sqrt{5}$.

3) $R(x) = 3$ then $\frac{3(x+2)}{x-2} = 3$ so $3(x+2) = 3x-6$ then $3x+6 = 3x-6$

then $6 = -6$ impossible (no solution).

Consider the system : $\begin{cases} 5x + 2y = 12000 \\ 3x + 6y = 24000 \end{cases}$

1) Is the couple (1000; 3500) the solution of this system?

2) A craftsman fabricates black pearls and golden pearls.

A bag containing 10 black pearls and 4 golden pearls are sold for 24000L.L.

A bag containing 3 black pearls and 6 golden pearls are sold equally for 24000L.L.

a) Write a system of two equations that translates the preceding text.

b) Calculate the price of one black pearl and the price of one golden pearl.

1) 5(1000)+2(3500) = 12000; 5000+7000 = 12000 so 12000 = 12000 (true).

3(1000)+6(3500) = 24000; 3000+21000 = 24000 so 24000 = 24000 (true).

Therefore (1000 ; 3500) is the solution of the system.

2) Let x be the price of a black pearl and y be that of a golden pearl.

a) $\begin{cases} 10x+4y=24000 \\ 3x+6y=24000 \end{cases}$

b) $\begin{cases} 10x+4y=24000.....\div(2) \\ 3x+6y=24000 \end{cases}$; $\begin{cases} 5x+2y=12000 \\ 3x+6y=24000 \end{cases}$ then by part (1) $x = 1000$ and $y = 3500$.

Therefore the price of a black pearl is 1000L.L. and that of a golden pearl is 3500L.L.

Consider in an orthonormal system x'Ox, y'Oy the points A(−2 ; 1) ; B(4 ; 1) and the straight line (D) of equation: $y = 2x + 5$.

1) Plot A and B.

2) Does A belong to (D)?

3) Plot (D).

4) Write the equation of straight line (AB).

5) The line (D) cuts x'Ox at E and y'Oy at F.
Find the coordinates of E and F, then calculate EF.

6) Write the equation of (D') passing through P(−1 ; 2) and parallel to (D).

7) Write the equation of (U) passing through A and perpendicular to (D).

8) Calculate the coordinates of I the midpoint of [BP].

9) Write the equation of (OA).

10) Determine m so that (D) is parallel to straight line (L) of equation $mx - (m - 1)y + 2 = 0$

11) Write the equation of line (V) passing through A and parallel to straight line (w): $y = 3$.

12) Write the equation of line (K) passing through B and perpendicular to straight line (R): $x = -3$.

1) Figure.

2) $y_A = 2x_A+5$; $1 = 2(-2)+5; 1 = -4+5$ so $1 = 1$, then A belongs to (D).

3) Figure.

4) (AB)//(x'x), since $Y_A = Y_B = 1$. So the equation of (AB) is $y = 1$.

5) $y_E = 0$ so $0 = 2x_E+5$; $-2x_E = 5$ then $x_E = \frac{-5}{2}$ therefore $E(\frac{-5}{2}\ ; 0)$.

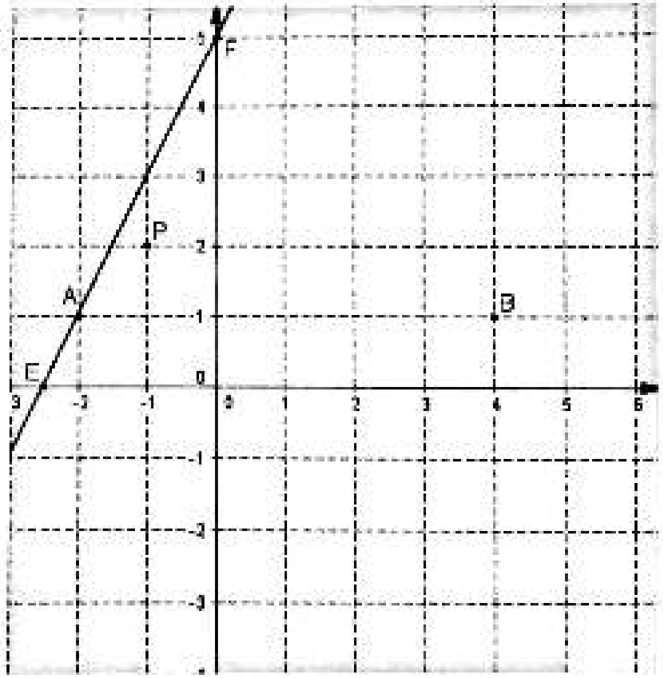
$x_F = 0$ so $y_F = 2(0)+5 = 0+5 = 5$ then F(0 ; 5).

$$EF = \sqrt{(X_E - X_F)^2 + (Y_E - Y_F)^2} = \sqrt{\left(\frac{-5}{2} - 0\right)^2 + (0-5)^2} = \sqrt{\frac{25}{4} + 25} = \frac{5\sqrt{5}}{2}.$$

6) $a_{(D')} = a_{(D)} = 2$ so $y_P = 2x_P+b$; $2 = 2(-1)+b$; $2+2 = b$ then $b = 4$ therefore (D'): $y = 2x+4$.

7) $a_{(U)} \times a_{(D)} = -1$; $a_{(U)} = \frac{-1}{2}$ so $y_A = \frac{-1}{2}x_A + b$;

$1 = \frac{-1}{2}(-2)+b$; so $b=0$ then (U): $y = \frac{-1}{2}x$.



8) $X_I = \frac{X_B + X_P}{2} = \frac{4-1}{2} = \frac{3}{2}$ and

$Y_I = \frac{Y_B + Y_P}{2} = \frac{1+2}{2} = \frac{3}{2}$, so $I\left(\frac{3}{2};\frac{3}{2}\right)$.

9) $a_{(OA)} = \frac{Y_A - Y_O}{X_A - X_O} = \frac{1-0}{-2-0} = \frac{1}{-2} = -\frac{1}{2}$,

so (OA): $y = \frac{-1}{2}x$.

10) (L): $-(m-1)y = -mx-2$;

$$y = \frac{-m}{-(m-1)}x - \frac{2}{-(m-1)} = \frac{m}{(m-1)}x + \frac{2}{(m-1)}$$

now $a_{(L)} = a_{(D)}$ then $\frac{m}{m-1} = 2$; $m = 2$.

11) (V): $y = b$ but (V) passes through A then (V): $y = 1$.

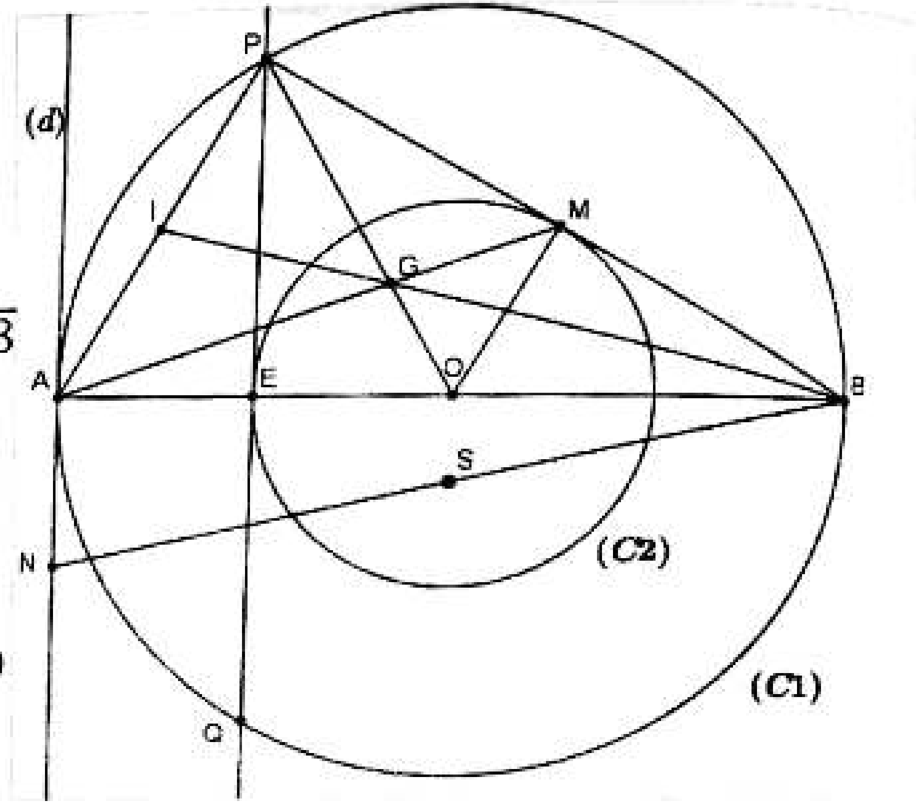
12) (K): $y = b$ but (K) passes through B then (K): $y = 1$.

- $C_1(O, 6cm)$ of diameter [AB] and $C_2(O, 3cm)$
- (OA) cuts (C_2) at E.
- The perpendicular to (OA) at E cuts (C_1) at P and Q.

1) Draw the figure.
2) a) Prove that AOP is an equilateral triangle.
   b) Deduce the nature of triangle APB.
   c) Calculate AP and PB.
3) Let M be the midpoint of [PB].
   a) Prove that M belongs to (C_2).
   b) Prove that (PB) is tangent to (C_2) at M.
4) Let I be the midpoint of [AP] and G be the point of intersection of (PO) and (AM).
   a) Prove that B, I and G are collinear.
   b) Calculate OG and GP.
5) Prove that the four points O, M, P and E belong to same circle of diameter to be determined.

1) See the figure.

2) a) As (PQ) perpendicular to (AO) and AE = AO-OE = 6-3=3 = OE, so (PE) is perpendicular bisector of [AO], so PA = PO and as PO = OA = R, then PA = PO = AO = 6cm, then triangle APO is equilateral.



b) $\widehat{APB} = 90^\circ$ (angle facing diameter), as $\widehat{PAB}=60^\circ$ then $\widehat{ABP} = 30^\circ$ then triangle ABP is semi-equilateral .

c) AP = R = 6cm and $PB = \frac{hyp\sqrt{3}}{2} = \frac{12\sqrt{3}}{2} = 6\sqrt{3}$

3) a) By midpoint theorem $OM = \frac{AP}{2} = 3cm = r$ so M belongs to (C_2).

b) (OM) perpendicular to (PB) ($\widehat{OMB} = \widehat{APB} = 90^\circ$ corresponding angles)

4) a) [AM] be the first median, [PO] second median so G is the centroid and [BI] the third median passing G, so B, I and G are collinear.

b) $OG = \frac{1}{3}OP = \frac{6}{3} = 2cm$ and GP = PO-OG = 6-2 = 4cm.

5) We have two right triangles POE and POM which are right triangles with common hypotenuse [PO].
So the four points O, M, P and E belong to same circle of diameter d = OP = 6cm.