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Intermediate Level - 7^{th} year

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Authors

A. MOARBES
K. ATTIEH
M. EL ASMAR
N. BADR
C. MERHEB
H. NASSAR
G. KARROUM

Editors

AL- AHLIA

Layout

Technical Department
AL-AHLIA

Press

AL-AHLIA Press

Distribution

Zouk Mikaël - Haret El Mir - Lebanon
Tel. 00 961 9 214144 - 5 - Cell : 00 961 71 315959
Fax : 00 961 9 213499 - P.O.Box : 369 Zouk Mikaël
www.ahliame.com - Email : al.ahlia@hotmail.com

Foreword

This book deals with the new program of the seventh year (intermediate cycle) with a new spirit : the individual building of concept, the formation of the student for better communication and critical thinking, the conservation of the link between the mathematics and real life situations.

Each chapter is formed of several parts.

- **The preparatory activities.** It is important not to neglect this part. It is short and accessible, enabling the introduction of a new concept, and sometimes even part of the lesson.
- **The course.** It is clear, simple and concise, hence respecting the new program. Certain essential results are highlighted so that the student may refer to them.
- **Exercises and problems.** A good number of exercises is proposed and presented in **three parts.**

  – **For testing the knowledge.** This part consists of the direct application exercises, thus helping the student verify whether he acquired the concepts or not.

  – **For seeking.** It consists of exercises demanding a thinking effort.

  – **The test.** The goal of this part is to control if the student assimilated well the studied concepts.

Notice that a large number of problems was chosen from real life situations familiar to the student. This is done in an effort to consolidate the link that exists between the daily life and mathematics.

We have two wishes. The first is to respect the time given for each chapter that appears in the table of contents. The second is the use of the calculator, which is demanded in the new program.

We hope that this work will be useful to the student of the 7^{th} year and that it will contribute in improving the teaching of mathematics.

We will be happy and thankful for any suggestion, criticism or advice.

The Authors

TABLE OF CONTENTS

1 GEOMETRY: The essential to start

Objectives

- Know the definition of a straight line, a semi-straight line (ray) and a segment.
- Know the definition of an angle, an acute angle, a right angle and an obtuse angle.
- Know the definition and the properties of the perpendicular bisector of a segment.
- Know the definition of the symmetry of a point with respect to a point and with respect to a line.

CHAPTER PLAN

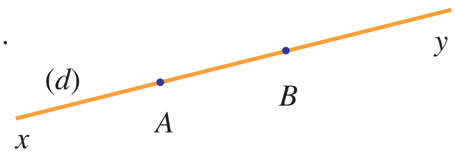
COURSE

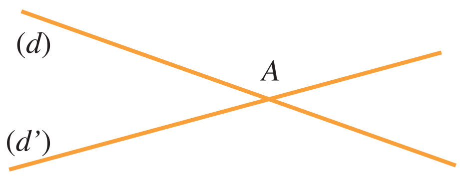
EXERCISES AND PROBLEMS

TEST

1 STRAIGHT LINE - SEMI-STRAIGHT LINE - SEGMENT OF A LINE

- **(*xy*)** is a **straight line**. It is also denoted by **(*AB*)** or by **(*d*)**.
- **[*Ay*)** and **[*Bx*)** are two **rays**.

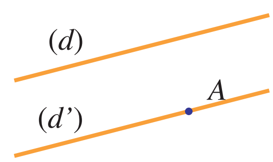


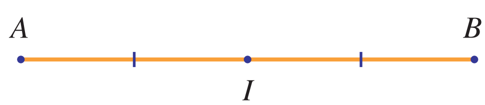


(*d*) and **(*d*′)** **intersect** (this means that they have one point in common). They are said to be **secant** (intersecting) at *A* or **concurrent**.

***A* is their intersection point.**

- **(*d*)** and **(*d*′)** **do not intersect**. They are **parallel**.
- **From point *A*, only one parallel (*d*′) can be drawn to (*d*).**



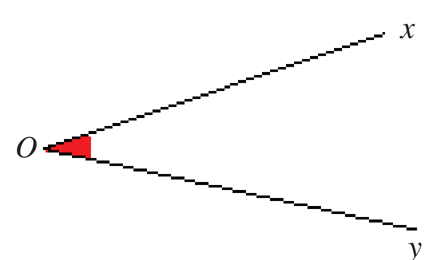


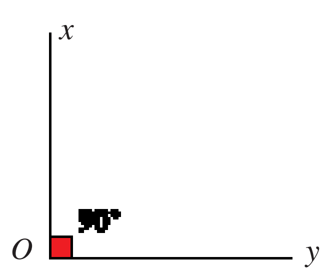
- **[*AB*]** is a **segment**.
  A and ***B*** are its **extremities**.
- If ***I*** is the **midpoint** of [*AB*], then ***IA* = *IB***.

2 ANGLES

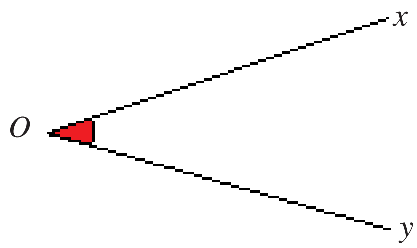
1°) • $\widehat{xOy}$ is an angle of **vertex *O***.
[*Ox*) and **[*Oy*)** are its **sides**.

- To **measure** it, the chosen unit is the **degree** (denoted by °).
- The **protractor** is the instrument used for measuring angles.





2°) $\widehat{xOy}$ is a right angle :
$\widehat{xOy}$ = 90°.
[*Ox*) is **perpendicular** to [*Oy*).

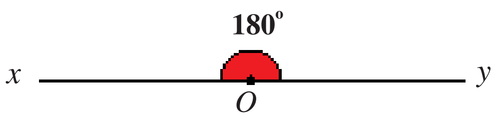


3°) $\widehat{xOy}$ is an acute angle : $0 < \widehat{xOy} < 90°$.

4°) [Ox) and [Oy) are two opposite rays.

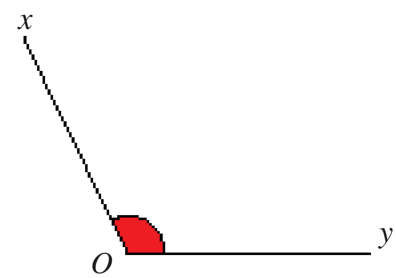
$\widehat{xOy}$ is a straight angle :

$\widehat{xOy} = 180°$.



5°) $\widehat{xOy}$ is an obtuse angle :

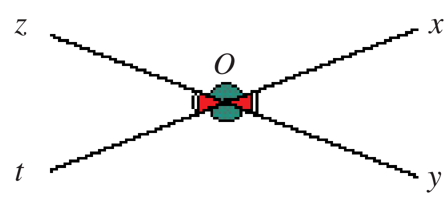
$90° < \widehat{xOy} < 180°$.



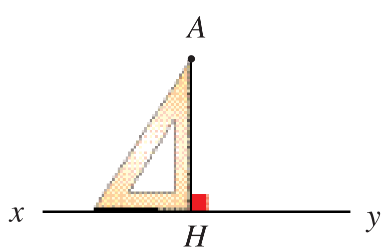
6°) $\widehat{xOy}$ and $\widehat{zOt}$ are vertically opposite angles, as well as $\widehat{zOx}$ and $\widehat{yOt}$.

We have :

$\widehat{xOy} = \widehat{zOt}$ and $\widehat{zOx} = \widehat{yOt}$.

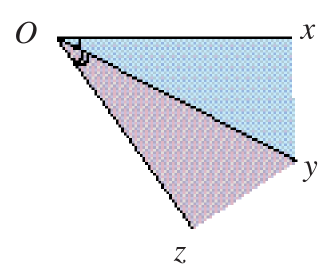


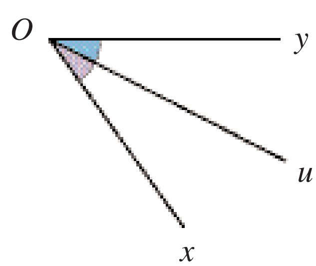
7°) AH is the distance from A to (xy).



8°) $\widehat{xOy}$ and **$\widehat{yOz}$** are **adjacent** since they have :

- **the same vertex O.**
- **a common side [Oy).**
- **[Ox) and [Oz) are situated on opposite sides of [Oy).**



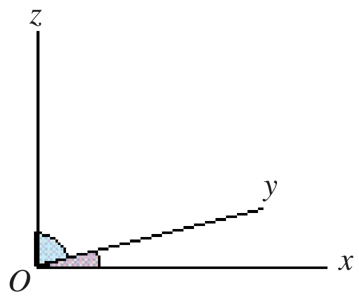


9°) [Ou) is the bisector of $\widehat{xOy}$:

it divides $\widehat{xOy}$ in two adjacent and equal angles.

$\widehat{xOu} = \widehat{uOy}$.

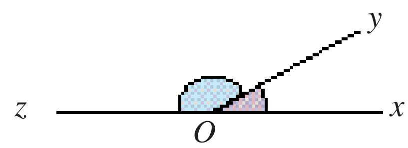
10°)



$\widehat{xOy}$ and $\widehat{yOz}$ are

adjacent and complementary :

$\widehat{xOy} + \widehat{yOz} = 90°$.



$\widehat{xOy}$ and $\widehat{yOz}$ are

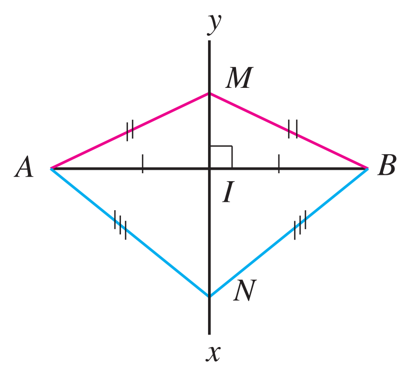
adjacent and supplementary :

$\widehat{xOy} + \widehat{yOz} = 180°$.

3 PERPENDICULAR BISECTOR OF A SEGMENT

- (xy) is the **perpendicular bisector** of $[AB]$. It is perpendicular to $[AB]$ at its midpoint I.
- **M and N belong to (xy) , the perpendicular bisector of $[AB]$, therefore :**

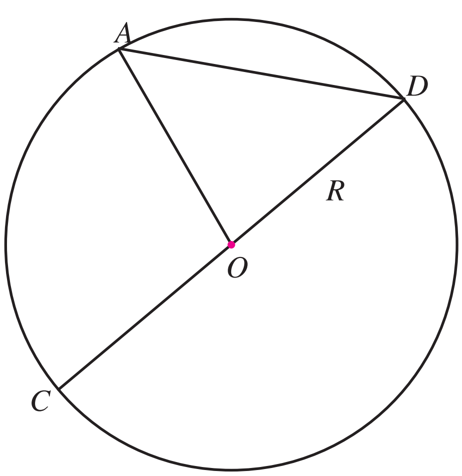
  $MA = MB$ and $NA = NB$.



4 CIRCLE

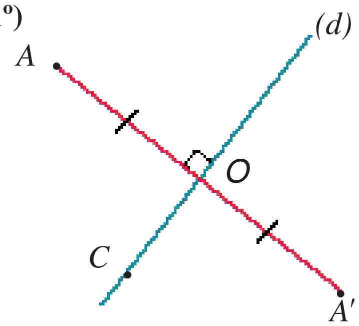
- O is the center of the circle.
- $[CD]$ is a **diameter** of the circle . C and D are two **diametrically opposite** points.
- $[AD]$ is a **chord** of the circle .
- $[OA]$ is a **radius** of the circle .

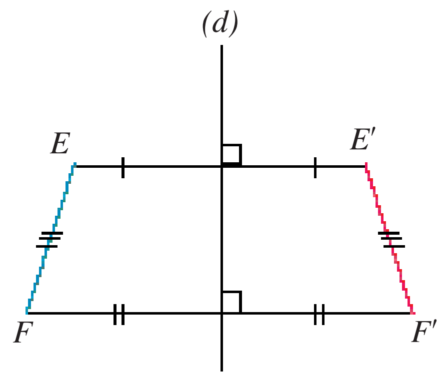
  $OA = OC = OD = R$
- A diameter is a chord that passes through the center of the circle. The measure of the diameter is double that of the radius : $CD = 2R$.



5 SYMMETRY

1°)



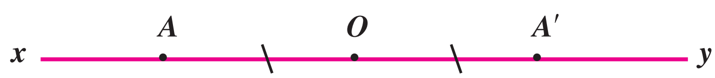


- A and A' are **symmetrical** to each other **with respect to** (d) if (d) **is the perpendicular bisector of** $[AA']$.
- If C is a point of (d), then it is the **symmetric of itself** with respect to (d).

The symmetric of segment $[EF]$ **with respect to** (d) is segment $[E'F']$ where (d) **is the perpendicular bisector of** $[EE']$ **and of** $[FF']$.

We have : $EF = E'F'$.

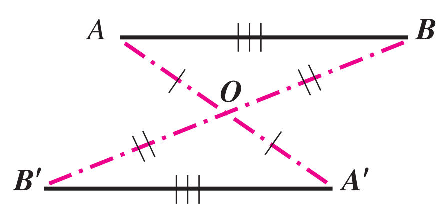
2°)



A' is the symmetric of A with respect to O if O is the midpoint of $[AA']$.

The symmetric of $[AB]$ with respect to O is $[A'B']$, where O is the midpoint of $[AA']$ and of $[BB']$.

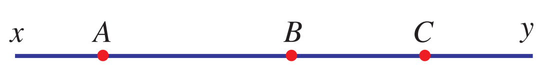
We have $AB = A'B'$.



Exercises and problems

For testing the knowledge

1 Answer the following questions using the figure below :



1°) Name the segments.
2°) Name the rays.
3°) Place point I , the midpoint of $[AB]$ and J, the midpoint of $[AC]$.
4°) Place point M on (xy) such that C is the midpoint of $[BM]$.
5°) Place point E on (xy) such that $AE = 2$ cm.
Is there another point F of (xy) situated at 2 cm from A?

2 On line (xy), place the points A, B, C and D in this order such that $AB = CD$.

1º) Show that $AC = BD$.

2º) Let I be the midpoint of $[BC]$. Show that I is the midpoint of $[AD]$.

3º) (d) is perpendicular to (xy) passing by I and E is a point of (d).

a) is $EB = EC$? Justify .

b) is $EB = ED$? Justify .

c) is $EA = ED$? Justify .

3 On a straight line (xy), place in order the points A, C, B and D with $AB = CD$.

1º) Show that $AC = BD$.

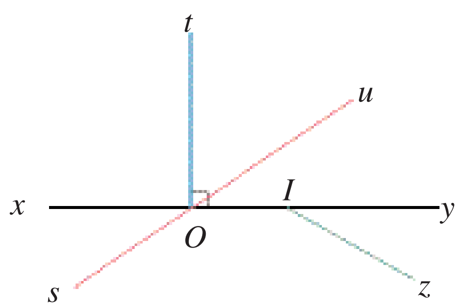
2º) Let I , J and K be the midpoints of $[AC]$, $[BC]$ and $[BD]$ respectively.

Show that : **a)** $AB = 2IJ$.

b) $JK = \frac{1}{2} CD$.

c) $AD + BC = 2IK$.

4



Using the figure above :

1º) Name a right angle and a straight angle.

2º) Name an acute angle and an obtuse angle.

3º) Name two vertically opposite angles.

4º) Name two adjacent angles.

5º) Name two adjacent complemen-tary angles and two adjacent supplementary angles.

6º) Answer by true or false :

a) $\widehat{uOt}$ and $\widehat{sOx}$ are vertically opposite.

b) $\widehat{yOu}$ and $\widehat{yOt}$ are adjacent complementary.

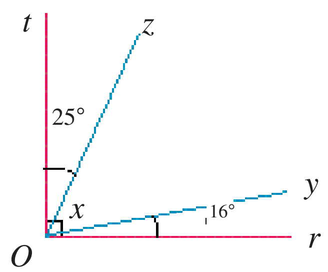
c) $\widehat{zIy}$ and $\widehat{yOu}$ are adjacent.

d) $\widehat{uOx}$ and $\widehat{xOs}$ are adjacent supplementary.

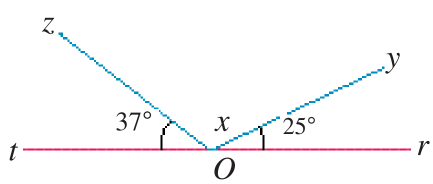
e) (xy) and (su) are perpendicular.

5 Calculate x in each of the following cases:

1°)



2°)



6 1°) Draw an angle $\widehat{xOy} = 80°$.

2°) Construct $[Oz)$ knowing that $[Ox)$ is the bisector of $\widehat{yOz}$.

3°) Construct $\widehat{xOt}$ adjacent and supplementary to $\widehat{xOy}$. Calculate $\widehat{xOt}$.

4°) $[Ou)$ is the ray opposite to $[Ox)$. Calculate $\widehat{yOu}$.

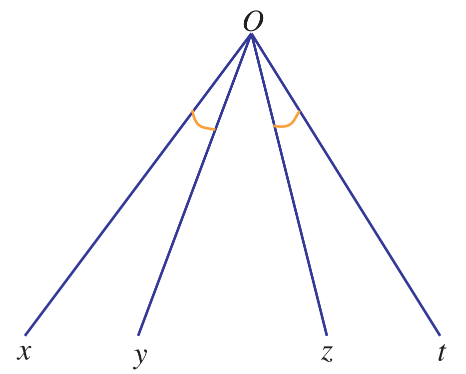
5°) What can be said about angles $\widehat{xOt}$ and $\widehat{yOu}$? Compare them.

7 $\widehat{mOn}$ and $\widehat{nOp}$ are two adjacent supplementary angles with $\widehat{mOn} = 50°$.

1°) Calculate $\widehat{nOp}$.

2°) $[Ox)$ and $[Oy)$ are the bisectors of $\widehat{mOn}$ and of $\widehat{nOp}$ respectively. Calculate $\widehat{xOy}$.

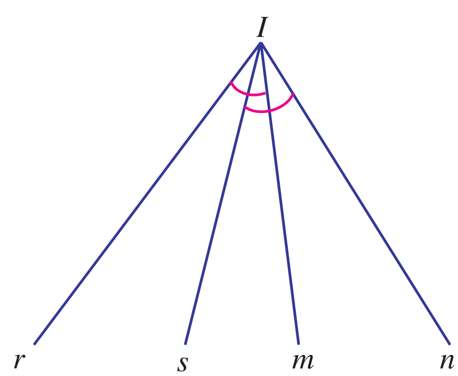
8 In the figure below, we have $\widehat{xOy} = \widehat{zOt}$.



1°) Show that $\widehat{xOz} = \widehat{yOt}$.

2°) Let $[Oy)$ be the bisector of angle $\widehat{yOz}$. Show that $[Ou)$ is the bisector of $\widehat{xOt}$.

9 Given that $\widehat{rIm} = \widehat{sIn}$.



1°) Show that $\widehat{rIs} = \widehat{mIn}$.

2°) Let $[Iu)$ be the bisector of angle $\widehat{rIn}$. Show that $[Iu)$ is the bisector of angle $\widehat{sIm}$.

10 $\widehat{xOy}$ and $\widehat{yOz}$ are two adjacent complementary angles. [Ou) and [Ov) are the bisectors of $\widehat{xOy}$ and $\widehat{yOz}$ respectively. What is the measure of $\widehat{uOv}$?

11 $\widehat{xOy}$ and $\widehat{yOz}$ are two adjacent supplementary angles. [Ou) and [Ov) are the bisectors of $\widehat{xOy}$ and $\widehat{yOz}$ respectively. What is the measure of $\widehat{uOv}$?

12 **1º)** Draw, using the ruler and the set square, the perpendicular bisector (xy) of [AB].

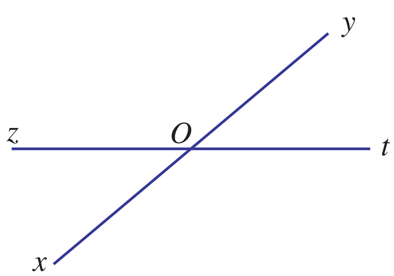
A ———————— B

2º) Place a point T on (xy). Compare TA and TB.

3º) Place a point F such that $FA = FB$.
Where is F found ?

For seeking

13 (xy) and (zt) intersect at O .

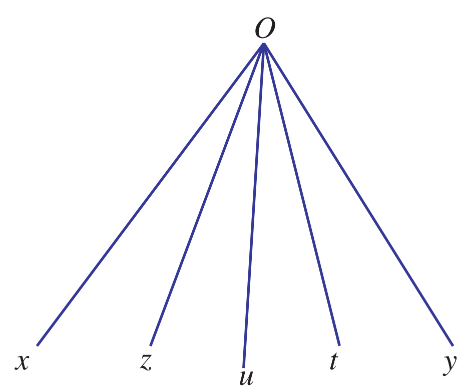


1º) Show that : **a)** $\widehat{zOy} = \widehat{xOt}$.

b) $\widehat{yOt} = \widehat{zOx}$.

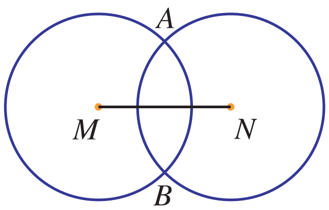
2º) Let [Ou) be the bisector of $\widehat{zOy}$, [Ov) is the opposite of [Ou) . Show that [Ov) is the bisector of $\widehat{xOt}$.

14 In the figure below, [Ou) is the bisector of the two angles $\widehat{zOt}$ and $\widehat{xOy}$.



1º) Show that $\widehat{xOz} = \widehat{tOy}$.

15 In the adjacent figure, the two circles of centers M and N have the same radius and are secant at A and B .

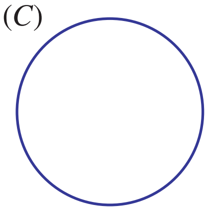


1°) Is (MN) the perpendicular bisector of $[AB]$? Justify.

2°) Which line is the perpendicular bisector of $[MN]$? Justify.

16 Ziad drew a circle and forgot to place its center I .

Can you help him place I ? Justify.



17 $[Ox)$ and $[Oy)$ are two semi-lines. A and B are two points of $[Ox)$, C and D are two points of $[Oy)$ with : $OA = OC$ and $OB = OD$.

1°) Show that $AB = CD$.

2°) Let I be the midpoint of $[AC]$. Show that (OI) is the perpendicular bisector of $[AC]$.

3°) Let J be the midpoint of $[BD]$. Show that (OJ) is the perpendicular bisector of $[BD]$.

4°) The perpendicular bisector of $[OA]$ cuts (OI) at H . Show that H is equidistant from the points O, A and C .

18 (d) and (d') intersect at I .
E and F are two points of (d) such that $IE = IF$.
A and B are two points of (d') such that $IA = IB$.

1°) Show that $AF = BE$ and $AE = BF$.

2°) The perpendicular bisectors of $[BE]$ and $[BF]$ meet at O . Show that O is equidistant from the three points B, E and F .

19 A is a point in the interior of the acute angle $\widehat{xOy}$.

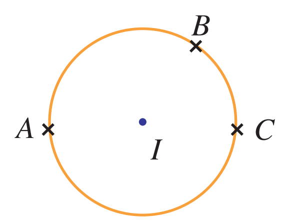
1°) Construct I and J , the symmetrics of A with respect to $[Ox)$ and (Oy) respectively.

2°) What do $[Oy)$ and $[Ox)$ represent for $[AJ]$ and $[AI]$?

3°) Show that $OI = OJ$.

TEST

1 Show that I, the center of the circle passing through A, B and C belongs to the perpendicular bisectors of $[AB]$, $[AC]$ and $[BC]$.

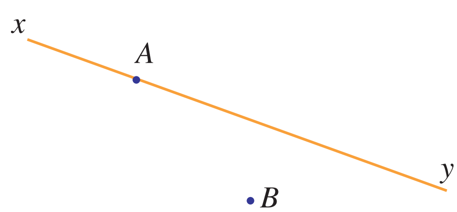


(3 points)

2 In the adjacent figure, A is a point of (xy) and B is a point that does not belong to (xy).

1º) Draw (uv) the perpendicular bisector of $[AB]$.

2º) Let O be the intersection point of (uv) and (xy) . Does the circle of center O and radius OA pass through B ? Justify.



(3 points)

3 Let I be a point on (d). A and B are two points of (d), symmetrical with respect to I . E and F are two points of (d), symmetrical with respect to I .

1º) Show that $AE = BF$.

2º) Show that $EB = AF$. **(4 points)**

2 ADDITION AND SUBTRACTION OF DECIMAL NUMBERS

Objectives

- Recognize a decimal number.
- Represent the decimal number on an axis.
- Recognize the opposite of a decimal.
- Compare two decimal numbers.
- Add two decimal numbers.
- Change the subtraction of two decimal numbers into addition.
- Perform calculations on decimal numbers.

CHAPTER PLAN

COURSE

EXERCISES AND PROBLEMS

TEST

1 DECIMAL NUMBERS

Definition

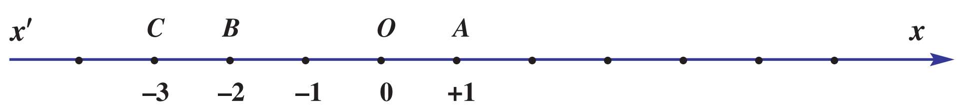
A decimal number is preceded by a + sign or a – sign.
• It is said to be positive if its sign is +
• It is said to be negative if its sign is –

Attention : Zero is the only number which is at the same time positive and negative.

EXAMPLES

• The number **+ 3.5** is positive.
• The numbers **– 5.7** and **– 3** are two negative numbers.
• The numbers **+ 2.6** ; **– 3** ; **+ 4** and **– 7** are decimal numbers.
• The numbers **+ 4.2** ; **– 5.11** and **+ 6** are decimal numbers.
• **+ 4.2** and **+ 6** are positive numbers.
• **– 5.11** is a negative decimal.

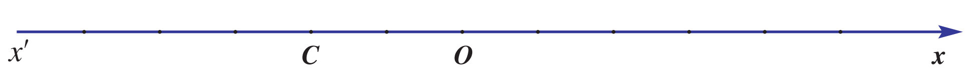
2 LOCATION ON A GRADUATED AXIS



The figure above represents a graduated straight line called **axis** ; *O* is its **origin**.

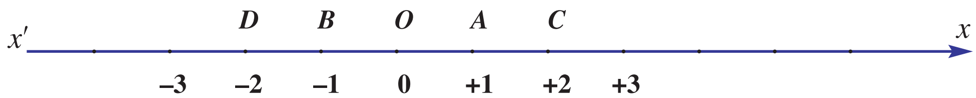
• *A* is the point of **abscissa + 1** and *B* is the point of **abscissa – 2** .
We denote them by $A(+\ 1)$ and $B(-\ 2)$.
• The sense from *O* to *x* is the positive sense, whereas from *O* to *x′* is the negative sense.
• Any number is represented on the axis by a point called the **image** of this number.
• $OA = 1$; 1 is **the distance to zero** from **+ 1** .
• $OC = 3$; 3 is **the distance to zero** from **– 3** .

Application 1



Using the axis above of origin *O* ,
1º) What is the abscissa of point *O*?
2º) What is the abscissa of point *C*?
3º) place the images of the numbers – 5 , + 3.5 and + 5 .
4º) place the points *E* and *F* whose distance from zero is 3 .

3 OPPOSITE DECIMAL NUMBERS



• The points A and B on the axis above are symmetrical with respect to the origin O ; their **abscissas** are said to be **opposite** .

• + 1 is the opposite of – 1 , and – 1 is the opposite of + 1 .

• **– 1 = opp (+ 1) and + 1 = opp (– 1) .**

• – 1 and + 1 are two opposite numbers .

• opp (0) = 0 .

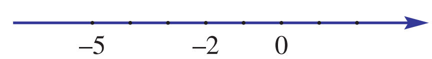
Application 2

What are the opposites of the following decimal numbers : + 3 ; – 7 ; + 2.1 ; 0 ; – 4.3 ; – 2 ; + 1.1 ?

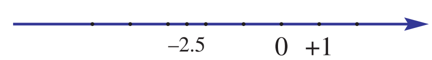
4 COMPARISON OF TWO DECIMAL NUMBERS

In order for two numbers to be compared, they should be placed on an axis. The one that is to the left is the smaller.

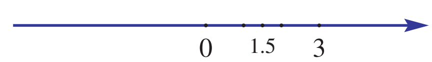
• – 5 is to the left of – 2 : – 5 < – 2



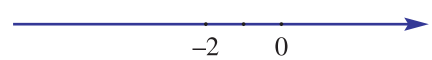
• – 2.5 is to the left of + 1 : – 2.5 < + 1



• + 1.5 is to the left of + 3 : + 1.5 < + 3



• – 2 is to the left of 0 : – 2 < 0



• 0 is to the left of + 1 : 0 < + 1 .

Application 3

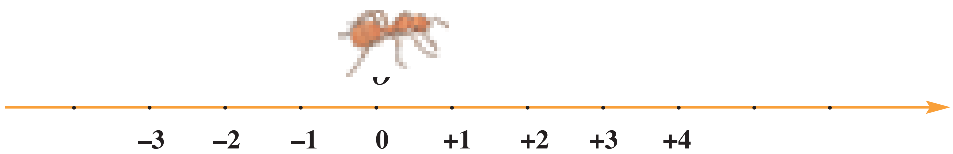
Complete by < or >

1º) – 7.1 ... + 2.8 **2º)** 0 ... – 15.3 **3º)** + 3 ... – 5

4º) + 2.15 ... + 5.01 **5º)** – 3.11 ... 0 **6º)** – 2.93 ... – 2.01

5 ADDITION OF DECIMAL NUMBERS

Activity



An ant is moving on the graduated axis above, where the chosen unit is the centimeter.

If it moves 3.5 cm to the right, it is said to move by **+ 3.5** .

If it moves 2.7 cm to the left, it is said to move by **– 2.7** .

The ant went from O to **+ 3** , then moved by **+ 2** . Will its position be on **+ 5** ? We write :
(+ 3) + (+ 2) = + 5 .

1°) Starting from O , the ant moves by **+ 4** then by **– 5** . What will be its position ? Complete :
(+ 4) + (– 5) = …

2°) The ant moves by **– 2** then by **– 3** .

Complete : (– 2) + (– 3) = …

1°) Sum of two decimal numbers having the same sign

To add two numbers having the same sign :

- add their distances from zero,
- the result is written with the sign of these two numbers.

EXAMPLES

1°) (+ 3) + (+ 5) = + 8 **2°)** (– 3) + (– 5) = – 8

2°) Sum of two decimal numbers having opposite signs

To add two numbers of opposite signs :

- calculate their difference,
- the result is written with the sign of the number having the longer distance from zero.

EXAMPLES

1°) (+ 8) + (– 3) = + 5 **2°)** (– 8) + (+ 3) = – 5

3°) (+ 3) + (– 3) = 0 **4°)** (+ 7) + opp (+ 7) = 0 .

Remark

The sum of two opposite numbers is equal to zero.

Application 4

Perform the following :

1°) (+ 3) + (+ 16.5) **2°)** (+ 4.25) + (– 6.75) **3°)** (– 2.5) + (– 12.5)

4°) (+ 13.9) + (– 13.9) **5°)** (+ 7.8) + (– 3.2) **6°)** 0 + (+ 14.27) .

6 SUBTRACTION OF DECIMAL NUMBERS

To subtract a decimal number from another, its opposite is added to the other.

EXAMPLES

1º) (+ 3) – (+ 2) = (+ 3) + opp (+ 2) = (+ 3) + (– 2) = + 1

2º) (– 5) – (– 3) = (– 5) + opp (– 3) = (– 5) + (+ 3) = – 2

3º) (+ 2) – (– 5) = (+ 2) + opp (– 5) = (+ 2) + (+ 5) = + 7

Application 5

Perform :

1º) (– 15.1) – (– 4.9) **2º)** (– 30) – (– 30) **3º)** (+ 5.3) – (– 3.2)

4º) (+ 35.5) – (– 35.5) **5º)** (– 20) – (+ 20) **6º)** (– 17.8) – (– 17.8)

7 METHODS OF CALCULATION

Calculate :

A = (– 5.2) + (+ 6.3) + (–14.5) + (+ 5.2) + (+ 8) + (– 30.4).

1st method

A = (– 5.2) + (+ 6.3) + (– 14.5) + (+ 5.2) + (+ 8) + (– 30.4).

We group the opposite numbers.

A = (– 5.2) + (+ 5.2) + (+ 6.3) + (– 14.5) + (+ 8) + (– 30.4)

A = 0 + (+ 6.3) + (– 14.5) + (+ 8) + (– 30.4).

We group the numbers of the same sign.

A = (+ 6.3) + (+ 8) + (– 14.5) + (– 30.4).

We add the numbers of the same sign.

A = (+ 14.3) + (– 44.9).

A = – 30.6 .

2nd method

Start adding from left to right :

$A = (-5.2) + (+6.3) + (-14.5) + (+5.2) + (+8) + (-30.4).$

$A = (+1.1) + (-14.5) + (+5.2) + (+8) + (-30.4).$

$A = (-13.4) + (+5.2) + (+8) + (-30.4).$

$A = (-8.2) + (+8) + (-30.4).$

$A = (-0.2) + (-30.4).$

$A = -30.6\ .$

Application 6

Calculate in two different methods :

$A = (+4.7) + (-3.8) + (-6.2) + (-4.7) + (+13.1) + (+3.8),$

$B = (-5.21) + (+8) + (-13) + (+13.7) + (-7) + (+5.21)\ .$

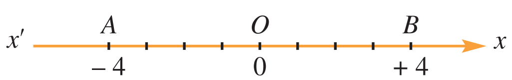
Exercises and problems

For testing the knowledge

1 Complete the following table.

a	opp (a)	distance to zero from a	distance to zero from opp (a)
– 5			
+ 49			
	– 7.2		
	+ 12.5		
+ 9.4			

2 Complete :



1º) – 4 and + 4 are the ... of the points *A* and *B* respectively.
2º) Since the abscissas of the points *A* and *B* are ... , then *A* and *B* are ... with respect to *O* ; point *O* is therefore the ... of [*AB*].
3º) + 4 is the distance to ... from – 4 .

3 Draw an axis $x'Ox$.
Plot on this axis the points $A(+1)$, $B(+4)$, $C(-5)$, $D(+2.5)$, $E(-2.5)$ and $F(-4)$.

a) What is the midpoint of [*BF*] ? the midpoint of [*DE*]?

b) Find *OB*, *OC* and *EF*.

4 Arrange from the least to the greatest.

a) -31.4 ; -27 ; -31.14 ; -3.1 ; -31.04 ; -31.42.

b) -19 ; -3.13 ; -19.5 ; -19.51 ; -3.01 ; -3.10.

5 Calculate.

$(+5) + (+7)$	$(-13) + (-14)$	$(-9) + (+5)$
$(+14.2) + (+3.4)$	$(-15.2) + (+10)$	$(-7.1) + (+9.4)$
$(-13.2) + (+13.2)$	$(-7.3) + (-1)$	$(+0.7) + (+11.2)$
$(-10.05) + (+0.05)$	$(+12.02) + (-12.2)$	$(+4.08) + (-398)$

6 Perform.

$(+17) - (+18)$	$(+13) - (-19)$	$(-14) - (+15)$
$(-13.2) - (-15.1)$	$(-5) - (+5)$	$(-13.4) - (-13.4)$
$(+386) - (-12)$	$(+32) - (-582)$	$(-1234) - (-1624)$

7 Calculate using the fastest way:

$A = (+115.2) + (+4.3) + (-115.2)$.

$B = (-2123.5) + (+5.1) + (+2123.5)$.

$C = (-17.2) + (-3) + (+15) + (+17.2) + (-12)$.

$D = (+12) + (-12) + (-9) + (+7)$.

8 Perform.

$A = (+5) + (-6) + (+10) + (-7) + (-5) + (+11)$.

$B = (-11123.6) + (+10) + (+1) + (+11123.6) + (-10)$.

$C = (-5.2) + (+3.02) + (+5.02) + (-3.2) + (+5.2)$.

$D = (+3.5) + (-6.2) - (-3)$.

$E = (-5.2) - (-3.5) + (-5.5)$.

$F = (-3.4) - (+3.4) - (-5.1)$.

$G = (-3.2) - (-4.2) + (-5.2) + (-6.3) + (-3.4)$.

$H = (-5.2) - (-3) - (-7.8) + (+0.5) + (-0.7)$.

9 Answer by true or false.

1º) Any natural number is a decimal number.

2º) 0 is not a decimal.

3º) (– 5) and (+ 7) are two opposite decimal numbers.

4º) (– 15) and (+ 15) are two numbers of opposite signs.

5º) (– 7) and (+ 7) are two opposite numbers.

6º) A number is always greater than its opposite.

7º) The sum of two numbers having opposite signs is zero.

8º) The sum of two opposite numbers is zero.

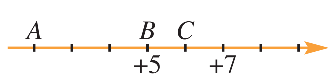
9º) The sum of two numbers having the same sign is always positive.

10º) The sum of two numbers having opposite signs is negative.

11º) The opposite of a number is negative.

12º) – 13.4 is less than – 12.4.

10 Choose the best answer.

Nº	**Questions**	**Answers**		
		a	**b**	**c**
1	(– 4) – (– 5) =	– 9	1	+ 9
2	(+ 7) – (+ 2.3) + (– 7) =	+ 2.3	+ 16.3	– 2.3
3	The abscissa of point *A* is 	– 1	0	+ 2
4	– 3.2 is less than	– 3.23	– 3.1	– 3.21

For seeking

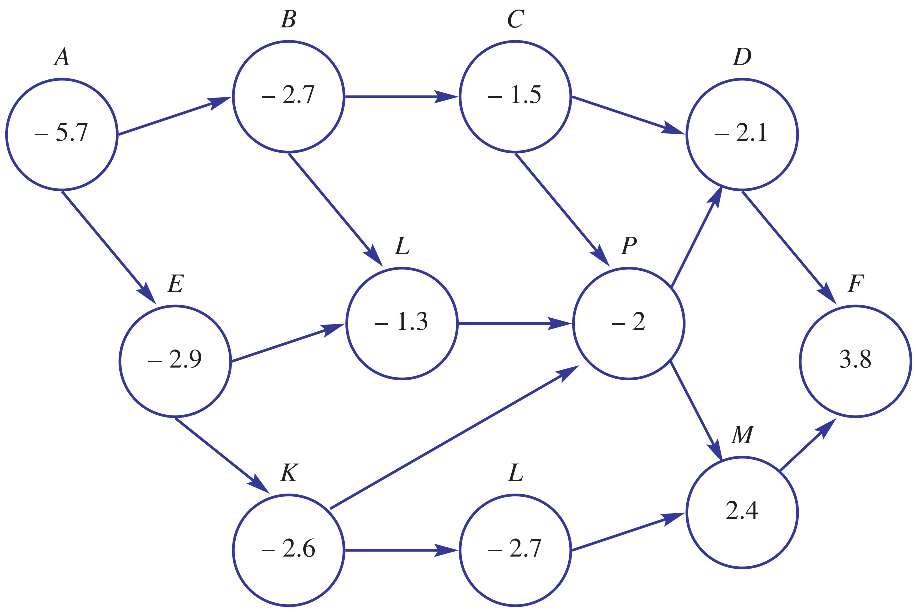
11 Given : $a = (-15) + (-3) - (-5)$,
$b = (-6) + (-4) + (+6)$ and
$c = (-5.1) - (-4.1) - (-6.3)$

Calculate : a, b, c, $a - b - c$, $a - (b - c)$ and opp $(a - b + c)$.

12 Calculate A , B and C then arrange them in increasing order.

A = $(-3.2) + (-5.1)$, B = $(+9.1) - (+17.2)$, C = $(-2.1) - (-4.5) + (-3.2)$.

13 Find the path from A to F , always passing by a greater decimal number :



14 Find a negative decimal number x knowing that :

- x is written with four digits,
- these digits are 1, 3, 4 and 8,
- a digit is used only once,
- $-3.484 < x < -3.471$.

15 In a game, a question is asked to each member of two teams.

Each player has the right to try twice and the following scores were obtained :

- right answer at the first try + 0.5 ,
- right answer at the second try + 0.2 ,
- wrong answer at the second try – 0.3 .

The results obtained by the members of the two teams are recorded in the following table :

1st team

Name	Answer
Walid	correct 1st try
Ziad	correct 2nd try
Rabih	wrong 2nd try
Kamal	correct 1st try

2nd team

Name	Answer
Zeina	wrong 2nd try
Nadine	correct 2nd try
Nadia	correct 1st try
Leyla	correct 1st try

1º) Write the score of each player.

2º) Write the score of each team. Which team won ?

16 **1º)** On an axis $x'x$ of origin O , locate point E of abscissa – 3 .

2º) What are the abscissas of the points of this axis whose distance from E is 4 ?

TEST

1 Calculate using the fastest way. **(6 points)**

a) (+ 9.4) + (+ 4.6) + (+ 1.13) + (+ 3.4).

b) (+ 45.12) + (– 8.5) – (+ 14.12) + (+ 1.5).

c) (– 3.5) + (+ 5) + (– 8) – (– 3.5) + (+ 14) – (+ 19).

2 Calculate starting from left to right. **(2 points)**

(– 2.2) + (– 3.4) – (– 5.7) – (+ 8.2).

3 Arrange in increasing order the following numbers. **(2 points)**

– 2.1 ; – 2.03 ; – 2.13 ; – 2.8 ; – 2.73.

4 During a whole week, Sami wrote each morning the evolution of the temperature in °C, with respect to the preceding day. **(5 points)**

Mon → Tues → Wed → Thur → Fri → Sat → Sun

+ 2   – 4   + 1   – 3   – 1   + 4

Knowing that the temperature of Sunday morning was 14 degrees, find the temperature of each day of the week.

5 These are 31 passengers in a bus.

At the first station, 9 passengers left the bus and 12 got on it.

At the second station, 6 passengers left and 2 got on it.

Find the new number of passengers on the bus. **(5 points)**

3 MULTIPLICATION AND DIVISION OF DECIMAL NUMBERS

Objective

Know and use the rule of multiplication and of division of two decimal numbers having the same sign and having opposite signs.

CHAPTER PLAN

COURSE

EXERCISES AND PROBLEMS

TEST

COURSE

1 MULTIPLICATION OF TWO DECIMAL NUMBERS

The product of two numbers having the **same sign** is a **positive** number.

$(+) \times (+) = (+)$ $(-) \times (-) = (+)$

EXAMPLES

$(+5) \times (+6) = +30$ $(-4) \times (-6) = +24$.

The product of two numbers having **opposite signs** is a **negative** number.

$(-) \times (+) = (-)$ $(+) \times (-) = (-)$

EXAMPLES

$(-3) \times (+4) = -12$ $(+2) \times (-8) = -16$.

Attention

$-(-3)$ means $(-1) \times (-3)$; therefore $-(-3) = +3$

$-(+5)$ means $(-1) \times (+5)$; therefore $-(+5) = -5$

$+(+2)$ means $(+1) \times (+2)$; therefore $+(+2) = +2$

$+(-3)$ means $(+1) \times (-3)$; therefore $+(-3) = -3$.

Application 1

1°) Calculate.

$(+3) \times (+7)$	$(+6) \times (-6)$	$-(-2.3)$
$(-5) \times (-8)$	$(-7) \times (+1)$	$-(+1.5)$
$0 \times (+5)$	$(-6) \times 0$	$+(-4)$.

2) Complete .

$(+5) \times \ldots = +20$	$(-4) \times \ldots = +12$	$\ldots \times (-6) = -42$
$(+3) \times \ldots = -15$	$(-5) \times \ldots = +35$	$\ldots \times (+8) = -40$
$\ldots \times (-5) = +60$	$\ldots \times (+5) = +5$	$\ldots \times (+10) = 0.$

2 CONVENTIONAL WRITING

A positive number is equal to its distance to zero.

$+3$ is written 3 ; $+1.5$ is written 1.5 .

EXAMPLES

1º) $+2 = 2$; $+11.8 = 11.8$.

2º) The equality $(+3) + (+5) = +8$ is written : $3 + 5 = 8$.

3º) $(+5) + (-3) = +2$ is written : $5 - 3 = 2$.

4º) $(+2) - (+6) = -4$ is written : $2 - 6 = -4$.

5º) $(+2) \times (+3) = +6$ is written : $2 \times 3 = 6$.

6º) $(-2) \times (-3) = +6$ is written : $-2 \times (-3) = 6$.

7º) $(-4) \times (+5) = -20$ is written : $-4 \times 5 = -20$.

8º) $(+6) \times (-5) = -30$ is written : $6 \times (-5) = -30$.

Application 2

Perform.

1º) $5 - 3 - 15 + 6 + 7 = \dots$

2º) $-(-24) = \dots$

3º) $3 \times (-5) + 2 \times (-7) - 4 \times (-6) = \dots$.

4º) $7 - (-2) + (-4) = \dots$

Remark :

The product of several numbers is :

• positive , if the number of negative numbers is even

• negative , if the number of negative numbers is odd.

EXAMPLES

1º) The product $-5 \times 6 \times (-213) \times (-421)$ is negative .

2º) The product $-7 \times (-628) \times 41 \times (-48) \times (-729)$ is positive .

Solved exercises

1°) **Calculate** $\boldsymbol{A = -4.5 - (-2) + (-3) + 1.2}$

$A = -4.5 + 2 - 3 + 1.2$

$A = -4.3$.

2°) **Calculate** $\boldsymbol{B = (8 - 7 \times 2) + (-5 - 1) - (-5)}$

$B = 8 - 14 + (-6) + 5$

$B = 8 - 14 - 6 + 5$

$B = -7$.

3°) **Calculate** $\boldsymbol{C = 12 - 5 + [4.8 + (9 - 7) \times 5] \times 2}$

$C = 12 - 5 + [4.8 + 2 \times 5] \times 2$

$C = 12 - 5 + [4.8 + 10] \times 2$

$C = 7 + 14.8 \times 2$

$C = 7 + 29.6$

$C = 36.6$.

3 DIVISION OF TWO DECIMAL NUMBERS

Divide the two numbers, then apply the following rule for the sign of their quotient :

$(+) \div (+) = (+)$; $(+) \div (-) = (-)$; $(-) \div (+) = (-)$; $(-) \div (-) = (+)$

EXAMPLES

- $3 \div 5 = 0.6$ is also written $\frac{3}{5} = 0.6$.
- $4 \div (-5) = -(4 \div 5) = -0.8$ is also written $\frac{4}{-5} = -\frac{4}{5} = -0.8$.
- $(-2) \div 8 = -(2 \div 8) = -0.25$ is also written $\frac{-2}{8} = -\frac{2}{8} = -0.25$.
- $(-6) \div (-10) = 6 \div 10 = 0.6$ is also written $\frac{-6}{-10} = \frac{6}{10} = 0.6$.

Attention

- $\frac{0}{5} = 0$; $\frac{0}{-3} = 0$.
- We cannot divide by 0 .

Application 3

Calculate .

1º) $(-7) \div (-20)$; **2º)** $(-5) \div 30$; **3º)** $\dfrac{-11}{4}$; **4º)** $\dfrac{-13}{-39}$.

4 ORDER OF CALCULATION

To perform operations :

- Start by calculating inside the parentheses
- Multiplication and division in the order of their appearance
- Addition and subtraction.

EXAMPLES

$A = 22 + 5 \times (15 + 2) - 28 \div (10 - 6)$
$A = 22 + 5 \times 17 - 28 \div 4$
$A = 22 + 85 - 7$
$A = 100$.

EXERCISES AND PROBLEMS

For testing the knowledge

1 Complete the following tables.

1º)

×	– 6	0	– 2	– 0.5
– 6				
0				
– 2	12			
– 0.5				

2º)

÷	– 6	12	0	– 13
2	– 3			
– 5				
4		3		
– 10				

2 Calculate .

1º) 0.25×4 **2º)** 0.2×5 **3º)** -0.125×8 **4º)** 2×0.5

5º) $0.0625 \times (-16)$ **6º)** -100×0.01 **7º)** $-2.5 \times (-0.4)$ **8º)** $25 \times (-0.04)$

3 Calculate.

1º) $15.2 - 15.4$ **2º)** $2.25 - 1$ **3º)** $15.2 \times (-15.4)$ **4º)** $2.25 \times (-1)$

5º) $\dfrac{-7}{5}$ **6º)** $\dfrac{1}{-2}$ **7º)** $\dfrac{0}{-3}$ **8º)** $\dfrac{-13}{10}$

4 Calculate *(you may use the calculator)*.

1º) 13.1×15.01 **2º)** -10×0.1 **3º)** -7.2×0

4º) -16.3×5 **5º)** $(-100) \times (-0.02)$ **6º)** $(-3.2) \times (-1)$

7º) $(-14.2) \times (-23)$ **8º)** $(-11.2) \times 19$ **9º)** $(-3) \div (-100)$

10º) $18 \div (-12)$ **11º)** $(-9) \div 25$ **12º)** $(-16) \div 20$

5 Calculate .

1º) $4 \times (5 + 6)$ **2º)** $4 \times (-5 + 6)$ **3º)** $-4 \times (-5 + 6)$

4º) $-4 \times (-5 - 6)$ **5º)** $4 \div (5 - 6)$ **6º)** $5 \div (7.2 - 7)$

6 Perform .

1º) $(-3) \times 2\times(-1) \times (-6) \times 2$ **2º)** $(-3) \times (-5) \times 3 \times 2 \times (-2)$

3º) $(-3) \times 2 \times (-1) \times 2 \times (-4) \times (-1)$ **4º)** $(-2.5) \times (-0.5) \times (-0.1)$

7 Complete by the convenient decimal.

1º) $-5.3 \times 16 = \ldots \times (-16)$ **2º)** $\dfrac{48}{\ldots\ldots} = -8$ **3º)** $\dfrac{-35}{\ldots\ldots} = 7$

4º) $24 \times (\ldots - 5) = 0$ **5º)** $-7.8 \times (-9.2) = \ldots \times 9.2$ **6º)** $\ldots\ldots \div (-7) = -9$

8 Perform.

1º) $-2 - (3.1 - 5) - (-3 + 2.3) - 9$ **2º)** $-3\,[4.1 - (3 + 5.2)] - (1 - 3)$

3º) $-[4 - (3 + 2)] - (2.3 - 1)$ **4º)** $+2\,(3 - 6) - [-1.5 - 1 + (-3 + 5)]$

9 Answer by true or false.

1º) If the product of two numbers is positive, then these two numbers are positive.
2º) The product of two numbers of opposite signs is negative.
3º) The product of two numbers is equal to the product of their opposites.
4º) $(-8) \div (-5) = -1.6$.
5º) $-9 - 5 = +45$.
6º) $-7 + 7 \times 2 = 0$.
7º) The product of a number by itself is positive.
8º) The product of seven negative numbers is positive.
9º) The product of four negative numbers is positive.
10º) $5 \times 2 \times [4 \times 3 + 2 \times (-6)] \times 10 = 0$.

10 Find the intruder .

$A = -4 \times (6 + 5)$ $\quad B = -2 \times (-11) \times 2$ $\quad C = -4 \times 6 + (-4) \times 5$
$D = 4 \times (-5 - 5) - 4$ $\quad E = -4 + 5 \times (-8)$ $\quad F = -4 - 5 \times 8$

For seeking

11 Calculate.

$A = 15 + 3.2 \times 2$ $\quad B = -16 + 5 \times 2$ $\quad C = 3.2 \times (-3) + 3 \div 5$
$D = 12 - 2 \div 4 + 5 \times 6$ $\quad E = 5 - 4 \times 2 + 5 \times (-3)$ $\quad F = 15 - 3 \div 15 - 5 \times (-2)$

12 Calculate.

$A = -5 \times (-6) - 3 \times [-5 - 3 \times (-4) + 6]$
$B = -3 + 3 \times [-6 - 7 \times 2 + 3 \times (-4)] - 5 \times (-5 - 6 \times 2)$
$C = -(-3 - 5 \times 4) - 5 \times (-4 - 3 \times 2) - 5 \times [-9 - 3 \times (-2)]$
$D = 3.25 - 5.25 \times [(3 - 5.2 \times 3) \times 5.1 - 5.1]$.

13 The product of two integers is -8.
Find all the possible values of these integers.

14 Calculate .

$A = -4.2 - [-5 - (8.3 + 16)] - (11.2 - 3)$
$B = -(2.4 - 1.5 - 5) - [12 - (-1 + 2.1 - 5)] + (1.2 - 7)$
$C = 12 - (-4.5 + 3.8) + [-5 - (2.3 - 10 - 4.7)]$
$D = -8 - (-7) \times (-3.1) + 5.2 \times (-8) - (-4.8)$
$E = (12 - 5 \times 8) \times (4 - 6 \times 3) - (-8.4) \times (3.2)$.

TEST

1 Calculate . **(4 points)**

$A = 8 \times 0.5 \times (-2) \times (-1)$

$B = -2.5 \times (-4) \times 10 \times 0.1$

$C = -7 \times (-6.1) \times (-0.1)$

$D = 0.1 \times (-10) + 3 \div (-5) - 0.2 \div (-10)$.

2 Complete : **(4.5 points)**

$-7 \times \ldots = 3.5$	$7 \div \ldots = 3.5$	$-7 \times \ldots = -3.5$
$+0.3 \times \ldots = 3$	$-0.3 \times \ldots = 3$	$\ldots \div (-10) = -5$
$-8 \times \ldots = -0.8$	$9 \times \ldots = -0.09$	$-1.3 \times \ldots = 13$

3 Pick a number, multiply it by -2 and add to the result the double of the chosen number.
Repeat the same procedure with another number.
What do you notice ? **(2.5 points)**

4 Perform . **(4.5 points)**

$A = -3.5 + 4 - (-9.5 + 1.5) - (-3 \times 2 - 0.2 \times 5)$

$B = (3.6 + 6.4) \times [2 + 3 \times (5 - 9) + 10 - 3]$

$C = 10.5 - 2 \times [(3 - 7.5) - 9 \times (-10 + 5.5)]$.

5 Find the number which is equal to (-10) times the double of 0.5. **(3 points)**

6 Below are two methods to calculate $A = 2 \times 5 + 7$: **(1.5 points)**

1st method	**2nd method**
$A = 10 + 7$	$A = 2 \times 12$
$A = 17$	$A = 24$

• Indicate the correct one.

4 LOCATION

Objectives

- Recognize the abscissa of a point on an axis.
- Define an orthogonal system of axes $x'x$ and $y'y$ and of origin O and know how to locate a point of the plane.
- Locate a point in a system knowing its coordinates.
- Recognize the four quadrants of the plane with respect to a system.

CHAPTER PLAN

COURSE

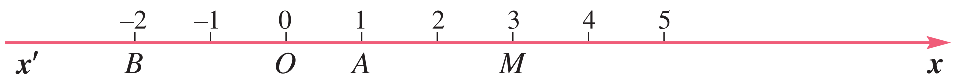
1 - Abscissa of a point
2 - Location of a point in a plane

EXERCISES AND PROBLEMS

TEST

1 ABSCISSA OF A POINT

$x'Ox$ is an axis of origin O.
For every number x, we associate a point M of this axis.
x is called **the abscissa** of this point ; we write : $\boldsymbol{x} = \overline{OM}$ and we read **«OM bar»** (algebraic measure).



EXAMPLES

- The abscissa of point M is 3 ; $\overline{OM} = 3$.
- The abscissa of point A is 1 ; $\overline{OA} = 1$.
- The abscissa of point B is – 2 ; $\overline{OB} = -2$.

Remarks :

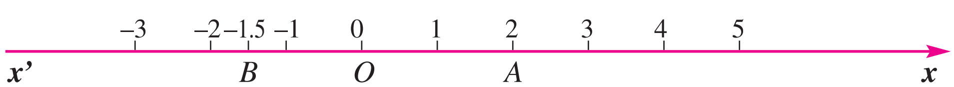
- The positive numbers are the abscissas of the points situated on $[Ox)$.
- The negative numbers are the abscissas of the points situated on $[Ox')$.
- The distance from A to B is 3. We write $AB = 3$ or $BA = 3$.

If we go from B to A on $[Ox)$, we write $\overline{BA} = +BA = +3$.

If we go from A to B on $[Ox')$, we write $\overline{AB} = -AB = -3$.

Application 1

1°) What are the abscissas of the points A and B on this axis ?
Complete $\overline{OA} = ...$; $\overline{OB} = ...$; $\overline{AB} = ...$



2°) Place the points C, D and E such that $\overline{OC} = 3.5$, $\overline{CD} = 1.5$ and $\overline{DE} = -6$.

3°) Deduce the abscissas of D and E.

2 LOCATION OF A POINT IN A PLANE

Activity

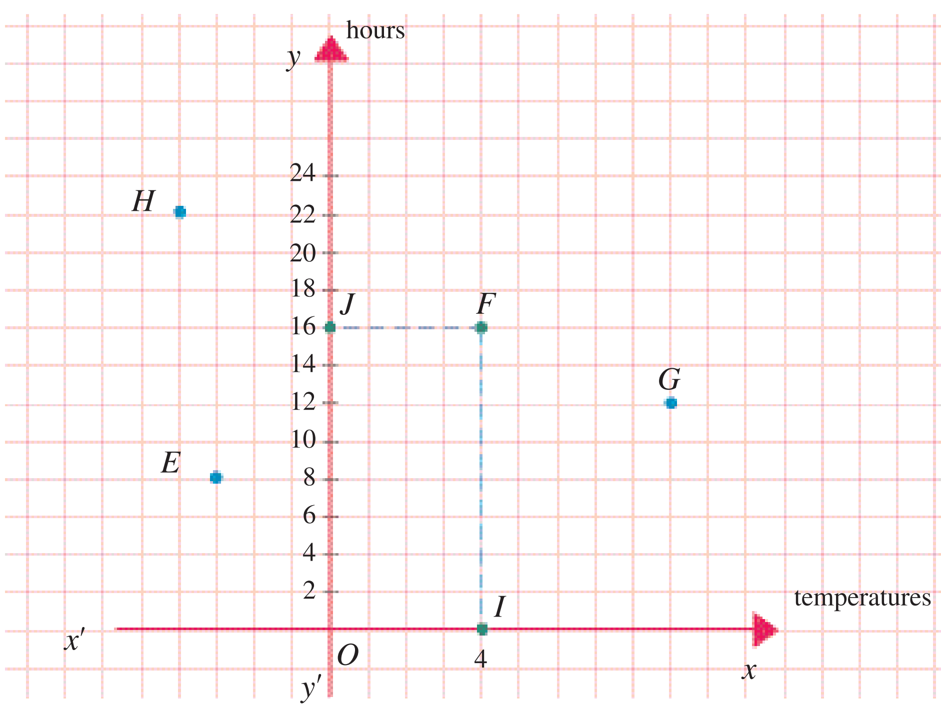
The following is the reading in degrees Celsius of the temperatures recorded during a winter day in the Cedars from 6 in the morning until midnight.

Hours	6	8	10	12	14	16	18	20	22	24
Temperature in degrees	– 8	– 3	2	9	8	4	0	– 2	– 4	– 5

1º) Represent in increasing order on the graduated axis $(x'x)$ below these different temperatures. We call O the point of this line where the temperature is equal to 0º.

2º) Let $(y'y)$ be a perpendicular axis to $(x'x)$ through O. On $(y'y)$ place the hours as shown below. Through the point I of $(x'x)$ that represents 4º , draw the parallel to $(y'y)$ and through the point J of $(y'y)$ that represents 16 hours, draw the parallel to $(x'x)$. These two parallels meet at F. This point indicates the time at which the temperature is 4º : It is denoted by F (4 ; 16).

Locate in a similar way the following points :
A (– 8 ; 6) ; B (2 ; 10) ; C (0 ; 8) ; D (–5 ; 24).



3°) What do the points E, G and H represent ?

Location

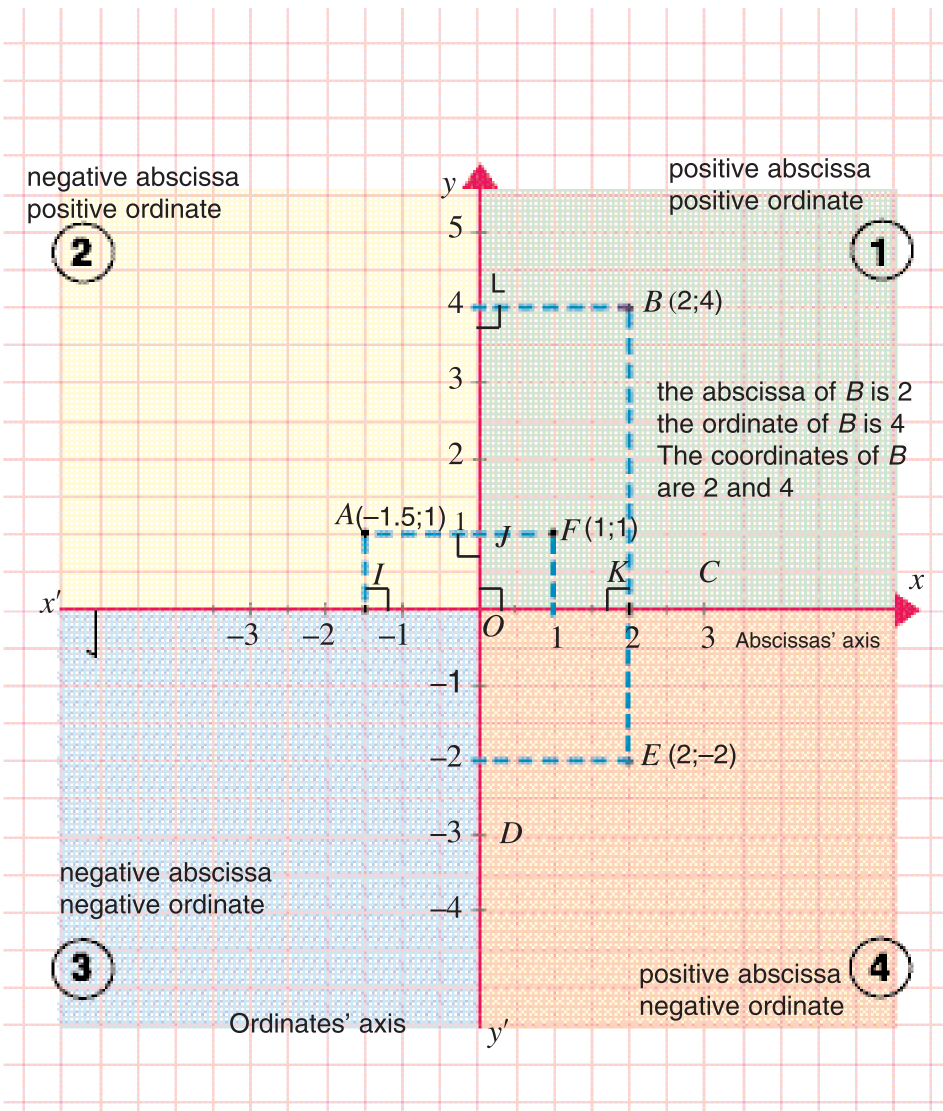
To locate a point in the plane, a **system** is chosen :

• an origin O

• two graduated axes $x'Ox$ and $y'Oy$ perpendicular at O .

If the **chosen units** are not the same, the system is said to be **orthogonal**.

If the units on both axes **are the same**, the system is **orthonormal**.



1º) In the preceding system, point A is located by the numbers –1.5 and 1. **–1.5** is its **abscissa** and **1** is its **ordinate**. **–1.5 and 1** are called the **coordinates of A.** It is denoted by A **(–1.5 ; 1)**.
To obtain the abscissa of A, draw from it a perpendicular to $x'Ox$, cutting it at I : I is the **orthogonal projection** of A on $x'x$, $\overline{OI}$ is the abscissa of A ; $\overline{OI} = -1.5$.

To obtain the ordinate of A , draw from it a perpendicular to $y'Oy$ cutting it a J : J is the **orthogonal projection** of A on $y'y$, $\overline{OJ}$ is the ordinate of A , $\overline{OJ} = 1$.

2º) The horizontal axis $x'Ox$ is called **the abscissas' axis**. The vertical axis $y'Oy$ is called **the ordinates' axis.**

3º) The system divides the plane in **four quadrants** numbered 1 , 2 , 3 and 4 . Point B, for example, is found in the first quadrant. Its coordinates are positive.

4º) All the points having a **null ordinate (zero)** are located on $\boldsymbol{x'Ox}$. For example, the points C (3 ; 0) and K (2 ; 0).
All the points having a **null abscissa (zero)** are located on $\boldsymbol{y'Oy}$. For example the points D (0 ; –3) and L (0 ; 4).

5º) The points having the **same abscissa** are located on a straight line **parallel to $\boldsymbol{y'Oy}$** . For example the points B (2 ; 4) ; E (2 ; –2) and K (2 ; 0).
The points having the **same ordinate** are located on a straight line **parallel to $\boldsymbol{x'Ox}$** . For example the points A (–1.5 ; 1) , F (1 ; 1) and J (0 ; 1).

Application 2

1º) Construct an orthonormal system ($x'Ox$, $y'Oy$) having as unit 1 cm .

2º) Locate in this system the points A (2 ; 1) ; B (–1 ; –1) ; C (0 ; 4) ; D (–3.5 ; 0) ; E (3 ; – 4.3) and F (–1.7 ; 2.1).

3º) Find the coordinates of points H and K such that (AH) and (BK) are parallel to $x'Ox$ and to $y'Oy$.

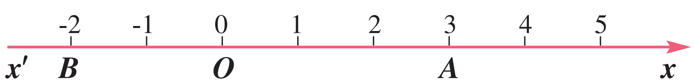
4º) Indicate the quadrants to which the following points belong :
L (–3 ; –5) , M (4.4 ; –2) and P (531 ; –192).

Exercises and problems

For testing the knowledge

1 Choose the best answer.

Using the axis $x'Ox$:



1º) the abscissa of point A is :

+ 1 ☐ + 3 ☐ – 2 ☐

2º) $\overline{AB}$ =

– 3 ☐ + 5 ☐ – 5 ☐

Using the system ($x'Ox$, $y'Oy$) below :

3º) (1 , 2) are the coordinates of point :

C ☐ A ☐ B ☐

4º) the coordinates of C are :

(1 ; –2) ☐ (1 ; 2) ☐

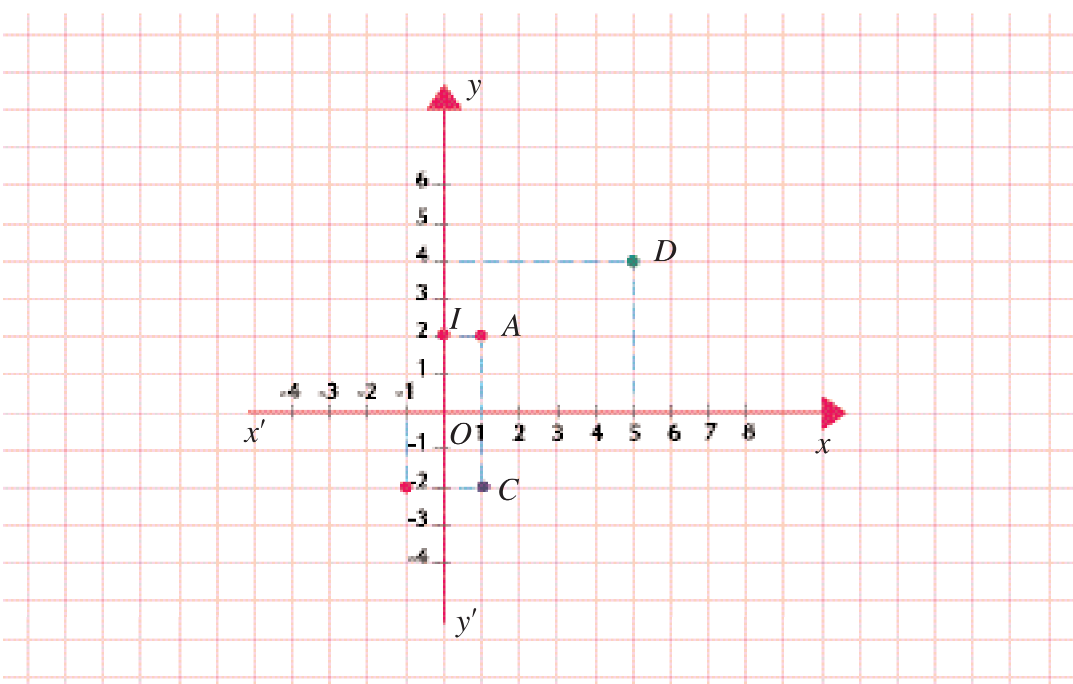
(–1 ; –2) ☐

5º) $\overline{OI}$ =

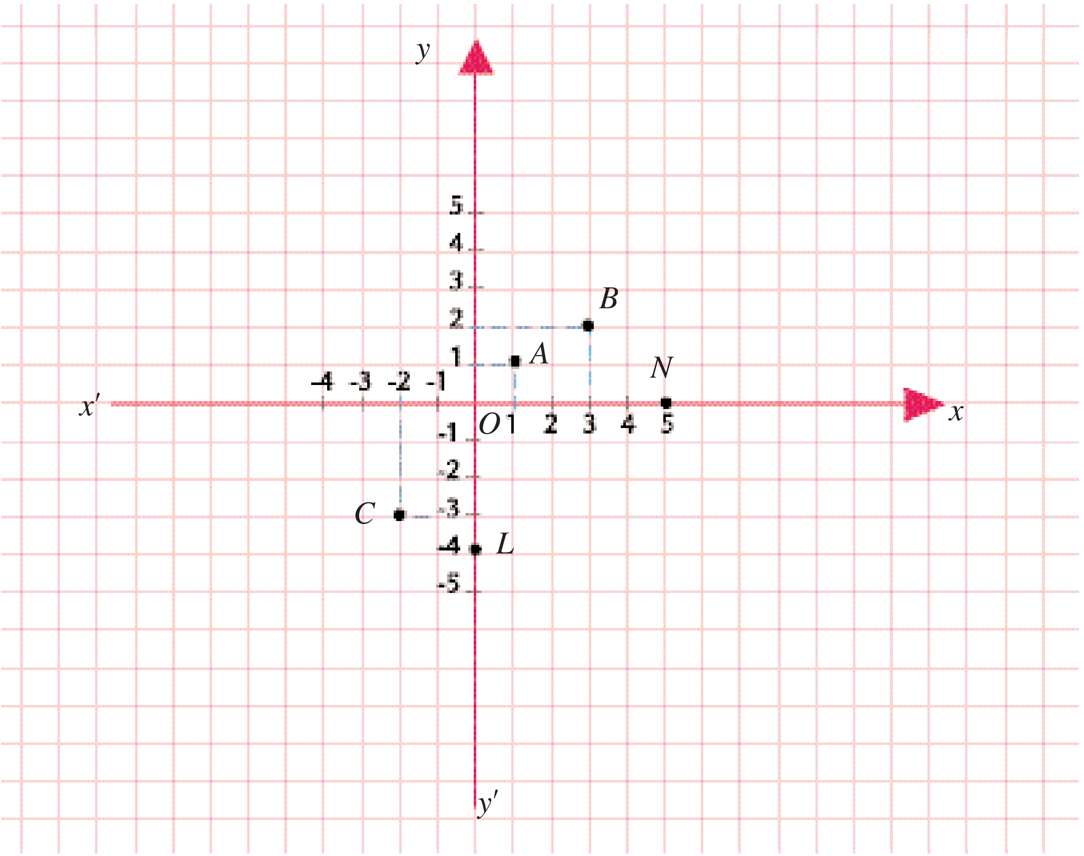
– 2 ☐ + 2 ☐ + 1 ☐

6º) the ordinate of D is :

+ 5 ☐ + 4 ☐ 0 ☐



2 $(x'Ox, y'Oy)$ is an orthonormal system.



1º) What are the coordinates of points A, B and C?

2º) Locate the points :

D (2 ; 0) ; E (0 ; –2) ; F (– 1 ; 2),

G (– 4 ; 2) ; H (–2 ; 1) ; K (–2 ; 5).

3º) What can you say about the points B, F and G? C, H and K?

4º) Locate the points I and J, the orthogonal projections of A and B on $x'x$.

Find the coordinates of I and J.

5º) Locate the points R and T, the orthogonal projections of C and K on $y'y$.

Find the coordinates of R and T.

6º) Locate point M knowing that its orthogonal projections on $x'x$ and $y'y$ are N and L respectively.

Find the coordinates of M.

3 Given the orthogonal system $(x'Ox\ ,\ y'Oy)$.

1º) Where are all the points having 0 as abscissa located ?

2º) Where are all the points having 0 as ordinate located ?

4 $(x'Ox\ ,\ y'Oy)$ is an orthonormal system.

State to which quadrant each of the following points belong :

$A\,(-1\ ;\ 2.5)$; $B\,(3\ ;\ 5.3)$; $C\,(-2\ ;\ 4)$; $D\,(-3.1\ ;\ -4.7)$;

$E\,(3\ ;\ -1.8)$; $F\,(5\ ;\ 6)$; $G\,(-2.7\ ;\ -16.4)$; $H\,(7.03\ ;\ -3)$.

5 Answer by true or false. $(x'Ox, y'Oy)$ is an orthonormal system.

1º) The abscissa of a point is always a positive number.

2º) The ordinate of a point is a decimal number.

3º) If the abscissas of A and B are 2 and 5 respectively, then $AB = -3$.

4º) The coordinates of the origin O of the system are $(0\ ,\ 0)$.

5º) The points $E\,(2\ ;\ 3)$ and $F\,(3\ ;\ 2)$ are coincident.

6º) Any point on $x'Ox$ has its abscissa equal to zero.

7º) The point $F\,(0\ ;\ 3)$ belongs to $y'Oy$.

8º) The point $H\,(-1\ ;\ 2)$ is in the second quadrant.

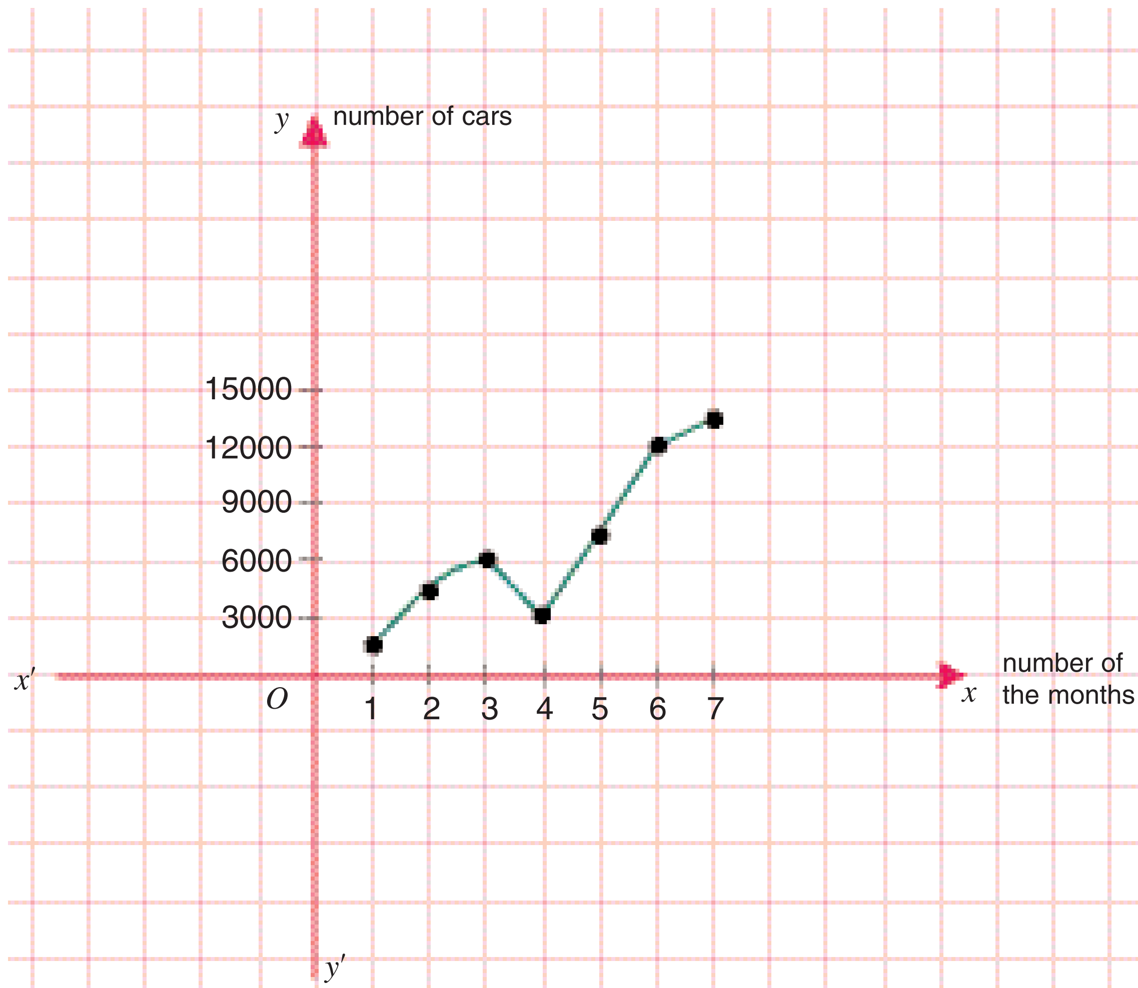
9º) Given the points $R\,(\ 2\ ;\ 3)$ and $T\,(2\ ;\ -5)$. The line (RT) is parallel to $x'Ox$.

10º) If J is the orthogonal projection on $x'Ox$ of a point I of the plane, then (IJ) is parallel to $y'Oy$.

For seeking

6 Without locating the points $A(2\ ;\ 6)$, $B(-3\ ;\ 6)$, $C(-3\ ;\ -5)$ and $D(2\ ;\ -5)$ in an orthonormal system $(x'Ox\ ,\ y'Oy)$, explain why the straight lines (AB) and (CD) are parallel to $x'x$.
What do you notice about (BC) , (DA) and $y'y$?

7 The curve below represents the sale of new cars during the first seven months of a year.



1º) Which is the month of the maximum sale ?

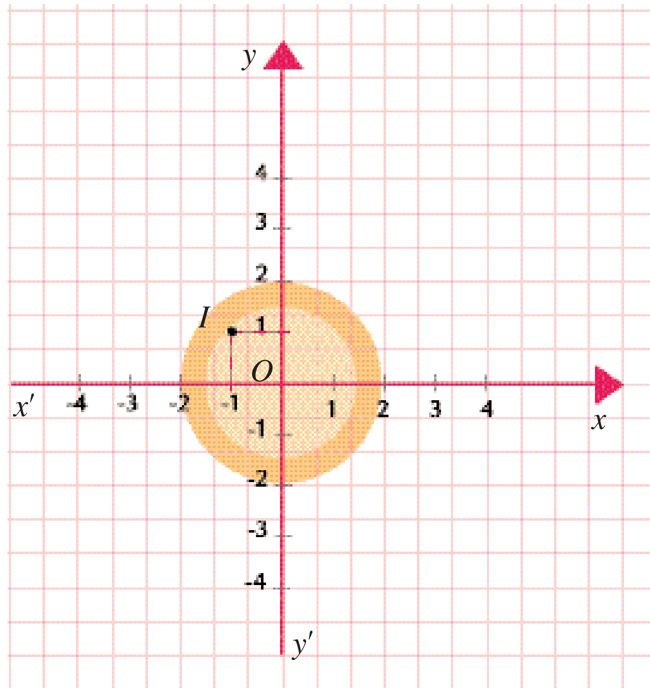
2º) Which is the month of the minimum sale ?

3º) Complete the following table :

Number of the month	1						7
Sale		4500		3000			

4º) What is the total number of cars sold between the fourth and the seventh months ?

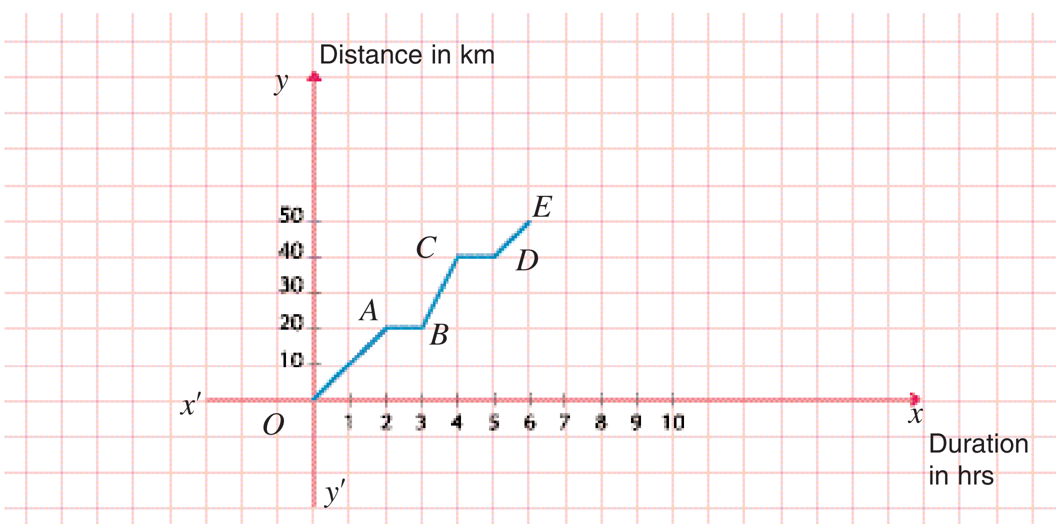
8 A game consists of throwing arrows on the target represented in the adjacent figure. If the arrow hits I , 20 points are won. 10 points are won if the arrow hits the interior of the small circle. 5 points are won if it hits the space between the two circles.
Walid, Sami and Kamal have thrown each three arrows that have fallen on the following points :
(0 ; 1) ; (–1 ; 3) ; (–2.5 ; 0) for Walid
(–2 ; 2) ; (2 ; 4) ; (–2 ; –2) for Sami
I ; (–1.5 ; 2) ; (–1.5 ; 3) for Kamal.



1º) Find the number of points gained by each.

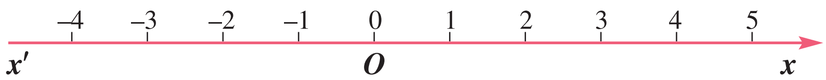
2º) Who is the winner ?

9 The graph below represents the trajectory of a cyclist who left from O for 6 hours.



Complete :
He reaches A at ...... hr after having traveled ...... km.
He rests for one hour then leaves from B at ...... hr.
He reaches C at .... hr after having traveled from B .... km.
He leaves D at ...... hr . To arrive at E at ...... hr, he still has to cover ...... km.

10 **1º)** $x'Ox$ is an axis of origin O.



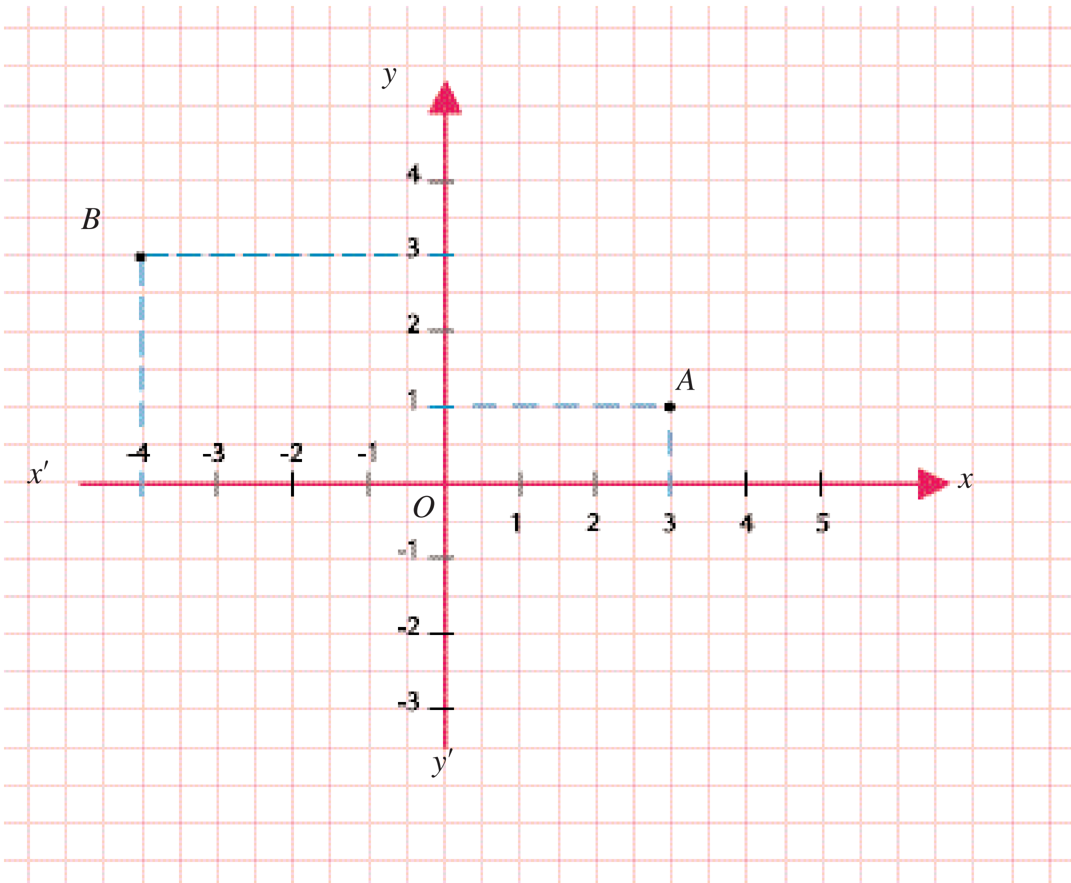
a) Place the points A, B, C and D of respective abscissas -4 , $+2$, $+4$ and -2.

b) Place point I , the midpoint of $[AB]$. What is its abscissa ?

Compare this abscissa with the half of the sum of the abscissas of A and B.

c) Calculate the abscissa of point J , the midpoint of $[CD]$.

2º) $(x'Ox \ , \ y'Oy)$ is an orthonormal system.



a) Find the coordinates of A and B.

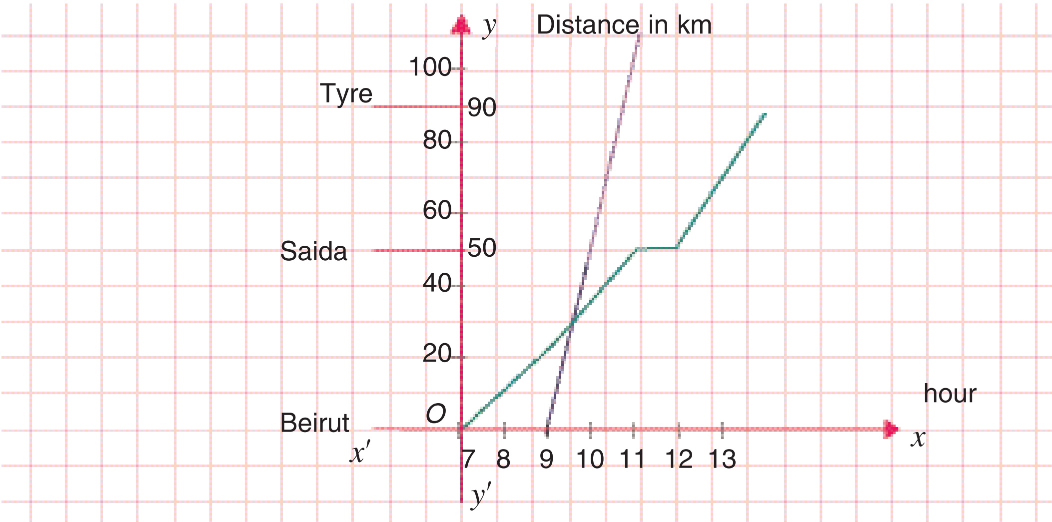
b) Place I , the midpoint of $[AB]$ and find its coordinates.

c) Compare the abscissa of I to the half of the sum of the abscissas of A and B. Do the same for the ordinates.

d) Place the points $C\ (3 \ ; -2)$ and $D\ (-1 \ ; 4)$.

e) Calculate the coordinates of J , the midpoint of $[CD]$. Locate J.

11 In the system below, the green line represents the trajectory of a cyclist and the purple line that of a driver.



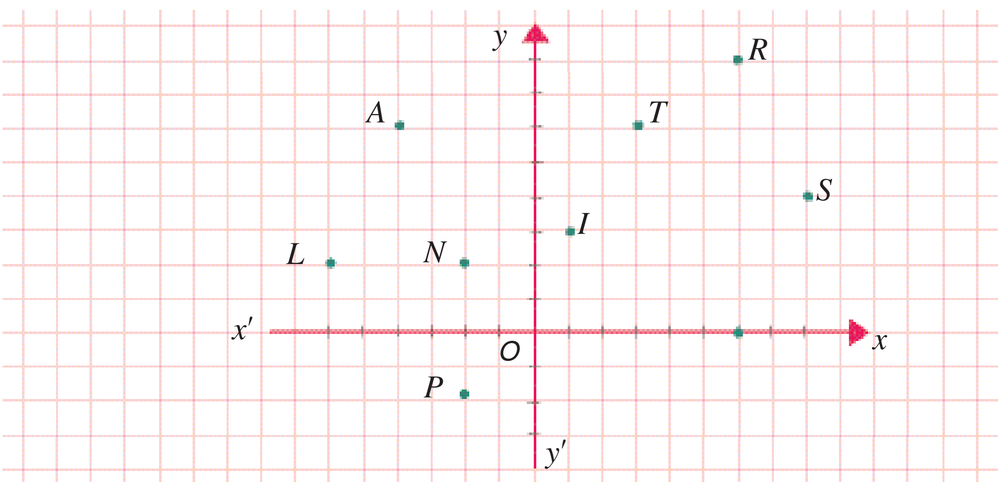
1°) At what time does the driver leave Beirut ?

2°) At what time does he pass by Saida ?

3°) At what time does the cyclist pass by Tyre ?

4°) At what time and at what distance from Beirut do the driver and the cyclist meet ?

12 Find a word made of five letters hidden among all the ones located in the system below.



Use the following information to find these letters :

First letter : its abscissa and its ordinate are equal (non-zero).
Second letter : its abscissa and its ordinate are null.
Third letter : its abscissa is the third of its ordinate.
Fourth letter : its ordinate is the opposite of its abscissa.
Fifth letter : its abscissa is the half of its ordinate.

TEST

1 **The mysterious word :**
($x'Ox$, $y'Oy$) is an orthonormal system where the chosen unit is 1 cm.

a) Join the following points :

First : A (–5 ; –1) → B (–5 ; 3) → C (–3 ; 3) → D (–3 ; 2) → E (–5 ; 2),

then : F (–2 ; 3) → G (– 2 ; – 1) → H (– 0.5 ; – 1),

after : I (1 ; 3) → J (1 ; –1) → K (3 ; –1) → L (3 ; 3),

finally : M (6 ; 3) → N (4 ; 3) → P (4 ; 1.5) → Q (6 ; 1.5) → R (6 ; –1) → S (4 ; –1).

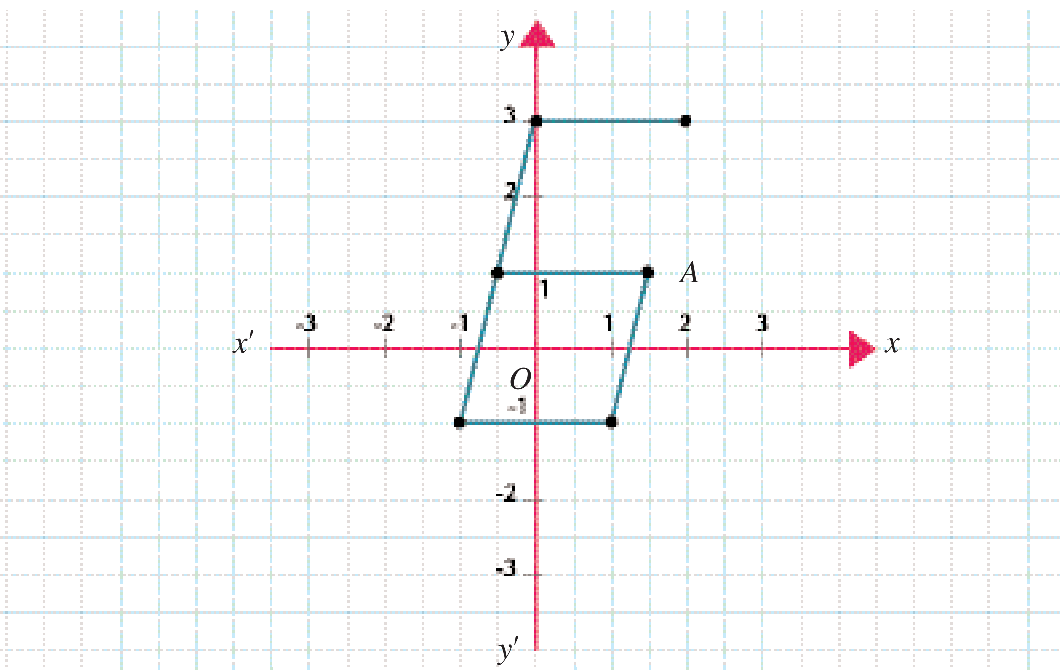
b) Find this mysterious word. **(6 points)**

2 **a)** In an orthonormal system ($x'Ox$, $y'Oy$) place the points A (–3 , –2) and B (2 ; 3).

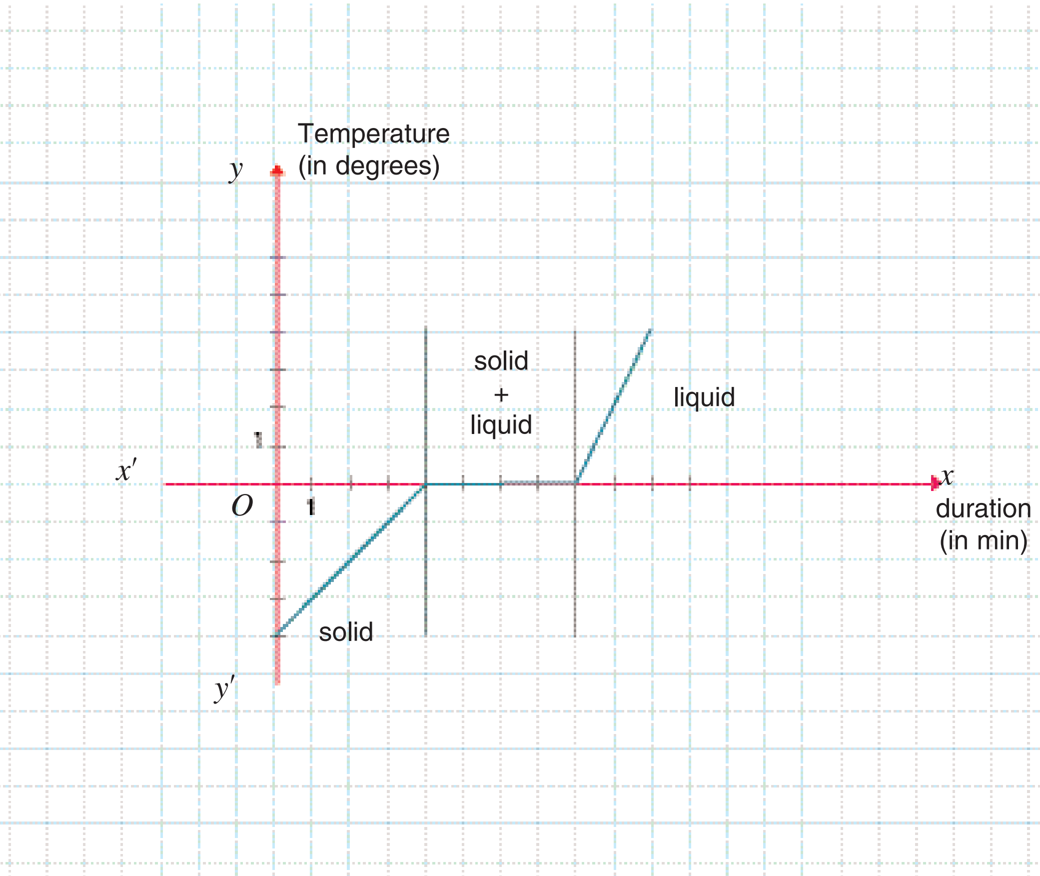
b) The straight line (AB) cuts $x'x$ at I and $y'y$ at J. Find th coordinates of I and J. **(3 points)**

c) Place on (AB) the point C of abscissa 1 and the point D having – 1 as ordinate.
Find the coordinates of C and D. **(5 points)**

3 In the system below, given the figure of the form ⊟.
Find the coordinates of the points that make this figure, starting from the point (2 ; 3) .
(3.5 points)



4 While heating ice cubes, the temperature was recorded every minute and the results are shown in the graph below.



1°) What is the initial temperature of the ice cubes ? **(1 point)**

2°) What is the temperature after three minutes ? after five minutes ? after ten minutes ? **(3 points)**

3°) After how many minutes will the ice cubes become totally liquid ? **(1.5point)**

5 STATISTICS

Objectives

- Take and organize data.
- Represent in a table the values and the frequencies.
- Calculate the relative frequency of each value.
- Represent a statistical distribution in a bar diagram and draw the frequency polygon.

CHAPTER PLAN

COURSE

1 - Vocabulary
1 - Population - Individual
2 - Characters
3 - Frequencies and relative frequencies
4 - Solved exercise

2 - Representation: bar diagram

EXERCISES AND PROBLEMS

TEST

1 VOCABULARY

Population - Individual

A statistical study consists of gathering and organizing information.

- The set on whom the study is done is called **population**. It may consist of people (students of a class, employees in an enterprise, inhabitants of a village, etc ...) of objects (cars, items, etc ...) of animals (hens, etc ...)
- Each element of the population is called **individual** : the student of a class, the employee, the inhabitant, the hen...

Character

The studied aspect of a population is called **character**. Two types of characters are distinguished :
• The characters that can be measured. They are said to be **quantitative** (height, weight, number of students, etc...). These characters have different **values**, also called **modalities.**
• The non-measurable characters. They are said to be **qualitative** (sex, color of eyes, kind of sport, etc...). There are no **values** for these characters, only **modalities**.

Frequencies and relative frequencies

Activity
The number of books read during the first semester by the students of a grade 7 class, is given by the following table :

Number of books read	0	1	2	3	4
Number of students	5	12	8	4	1

1º) What is the number of students of this class ?
(This number is called the **total frequency** of the class).

2º) **a)** What is the number of students that have read two books ?
(This number is called the **frequency of the value 2**).
What fraction of the class does it represent ?
(This number is the **relative frequency** of 2).
b) Calculate the frequency and the relative frequency of the students that have read only one book.

3º) What does the number 5 represent in this table ?
4º) **a)** What is the number of students that have read at least two books ?
b) What is the number of students that have read less than two books ?

Definitions

• The number of individuals of a population is called the **total frequency** of the **population**.

• The number of individuals that verifies a specific value of a character is called the **frequency of this value.**

• The ratio $\dfrac{\textbf{frequency of a value}}{\textbf{total frequency}}$ is called **the relative frequency** of this value.

Remarks :

• The sum of the frequencies of all the values is equal to the total frequency of the population.

• The relative frequency of a value or modality of a character is a number included between 0 and 1.

• The sum of the relative frequencies is equal to 1.

• The relative frequency may be expressed in percentage.

(The relative frequency $\frac{1}{5} = 0.2$ is expressed by 20%).

Solved exercise

A survey done on the students of a grade 7 class about their favorite hobby gave the following results :

Hobby	Sports	Movie	Television	Music	Computer
Frequency	5	8	4	3	10

• The studied population is : the set of the students of this class.

• The individual is : Each student of this class.

• The studied character is : the favorite hobby. It’s a qualitative character. The **modalities** of this character are : sport, movie, television, music and computer.

• The total frequency of this population is : $5 + 8 + 4 + 3 + 10 = 30$.

• Eight students have the movies as their favorite hobby. We say that the frequency corresponding to the «movies» is 8.

The preceding table, grouping all the different modalities of the character and their frequencies is called **table of frequencies.**

• The relative frequency of the «music» is : $\frac{3}{30} = \frac{1}{10} = 0.1$; that is 10%.

• The table below, grouping all the different modalities of the character with their relative frequencies is called **table of relative frequencies.**

Hobby	Sports	Movie	Television	Music	Computer
Relative frequency	$\frac{1}{6}$	$\frac{4}{15}$	$\frac{2}{15}$	$\frac{1}{10}$	$\frac{1}{3}$
Relative frequency in percentage	16.66%	26.66%	26.66%	13.33%	33.33%

We notice that : $\frac{1}{6} + \frac{4}{15} + \frac{2}{15} + \frac{1}{10} + \frac{1}{3} = 1$ or 100%.

Application 1

A study made on 100 families on the number of their children gave the following results :

Number of children	0	1	2	3	4	5
Frequency	5	15	40	25	12	3

1º) What is the studied character ? Give its nature.

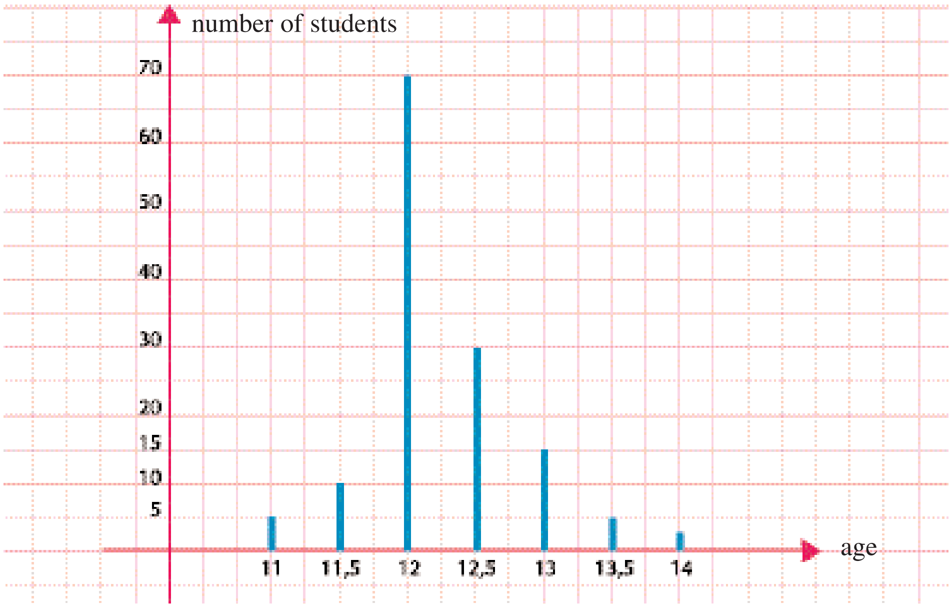
2º) What does the number 25 represent in this table ?

3º) What is the number of families that have more than three children ?

4º) What are the frequency and the relative frequency of the value «3» ? Give in percentage, the relative frequency of this value.

5º) Represent the relative frequencies of this study in a table.

Activity 2 REPRESENTATION



The graph above represents the distribution of the students of grade 7 according to their age.
It is called a **bar diagram** of the frequencies.

1°) Complete the following table.

Age	11	11.5	12	12.5	13	13.5	14
Number of students		10					

2°) What is the frequency of the age 12?

3°) What is **the total frequency** of the sections of grade 7 ?

4°) What is **the relative frequency** of the age 12?

5°) Join the extremities of the bars. The obtained broken line is called the **polygon of frequencies.**

Bar diagram

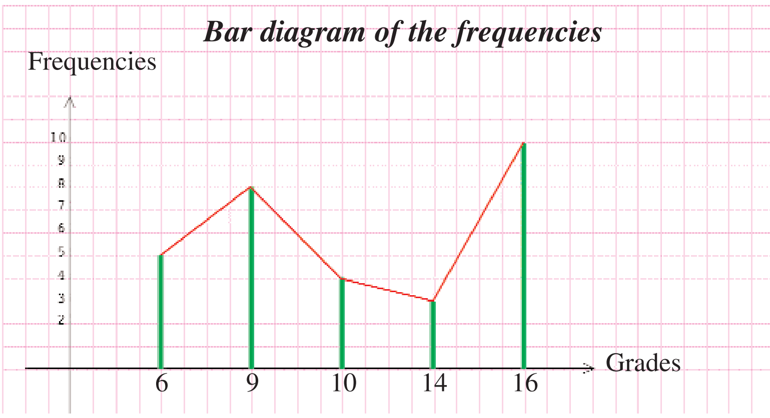
The obtained results of a study may be represented in the form of tables, as in the preceding examples. There are other methods too.

One of these methods is the **bar diagram**. We draw an orthogonal system where the abscissa axis represents the values of the studied character, and the ordinate axis represents their frequencies or their relative frequencies.

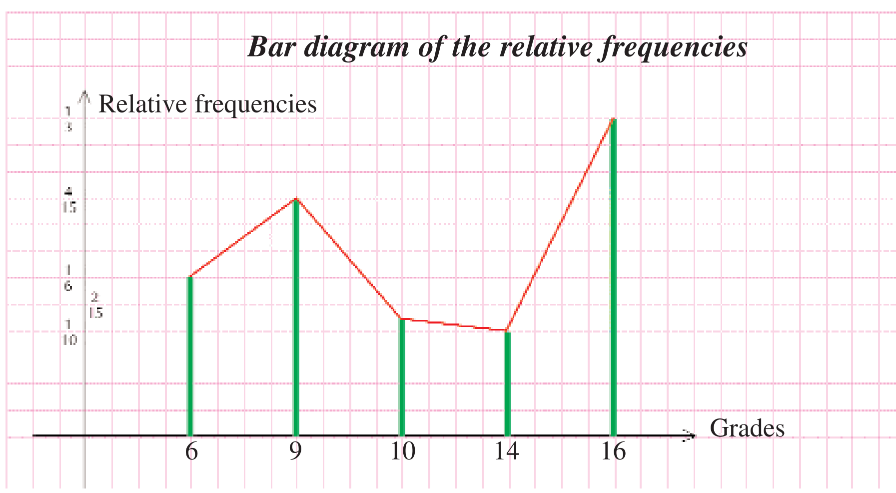
Below is a table showing the grades over 20 obtained on a test by the students of grade 7 .

Grade	6	9	10	14	16
Frequency	5	8	4	3	10
Relative frequency	$\frac{1}{6}$	$\frac{4}{15}$	$\frac{2}{15}$	$\frac{1}{10}$	$\frac{1}{3}$

The bar diagram representing the result of this study are the following :



The broken line joining the extremities of the bars is called the **frequency polygon.**



The broken line joining the extremities of the bars is called the **relative frequencies polygon.**

Application 2

Draw the frequency bar diagram of the study done in application 1.

EXERCISES AND PROBLEMS

For testing the knowledge

1 A survey done on the students of a class about their practiced activity gave the following :

Sport	Football	Basketball	Ping-Pong	Tennis	None
Frequency	8	10	12	6	4

1º) What is the population of this study ?
2º) What is the studied character ? Give its nature.
3º) Calculate the total frequency of this population.
4º) What does the number 12 represent in this table ?
5º) Construct the frequency bar diagram.
6º) Calculate the relative frequency in percentage of the basketball.
7º) Represent in a table the relative frequencies.

2 300 students of a college are divided in : 100 semi-boarding students, 150 day-scholar students and 50 boarding students.
1º) Represent this division in a table.
2º) Represent the division of the students in a bar diagram. Construct the frequency polygon.
3º) Calculate the percentage of the day-scholar students.
4º) Represent the relative frequency in the table.

3 Answer by true or false.
The following table represents the number of brothers and sisters of the students of a grade 7 class.

Number of brothers and sisters	0	1	2	3	4
Frequency	5	10	8	4	3

1º) The population is the set of the brothers ans sisters.
2º) The studied character is quantitative.
3º) There are three students who have three brothers and sisters.
4º) The total frequency of the class is 30.
5º) The frequency of the value «2» is 10.
6º) The relative frequency of the value «1» is $\frac{1}{3}$.
7º) 10% of the students of this class have four brothers and sisters.
8º) There are ten students in this class who have more than two brothers or sisters.

For seeking

4 In a maternity, the weighing of twenty newborns, expressed in kg, gave the following results :

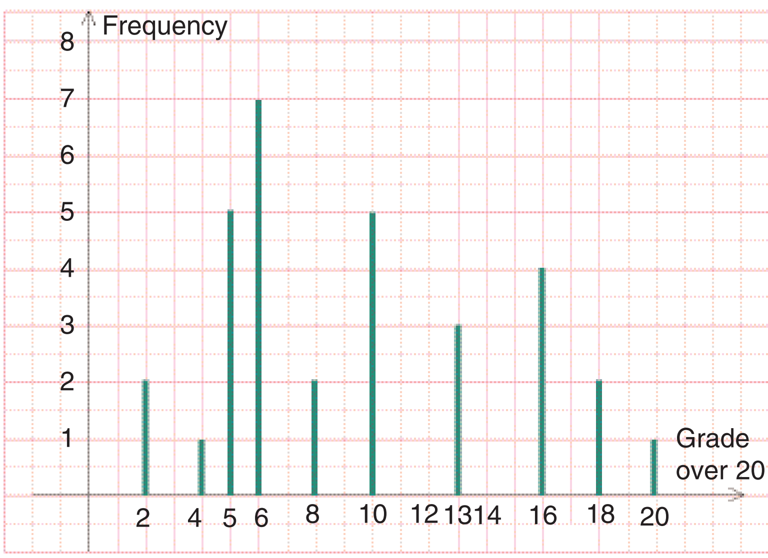
2.3	2.3	2.5	3.5	3.2	3	3	3.2
3.5	2.5	2.3	3	3	3.5	2.5	3.5
3	3.5	3	2.5				

1º) Represent these results in a table showing the frequencies and the relative frequencies in percentage.

2º) What is the most frequent weight ? the least frequent ?

3º) Represent the bar diagram of the percentages.
Construct the frequency polygon.

5 The bar diagram below represents the distribution of 32 test papers (the grades are over 20).



1º) Translate these results in a table.

2º) Give the frequencies of the grades 8 , 16 and 20.

3º) Calculate the percentage of the students having 10.

4º) Calculate the number of students having less than 10.

5º) Calculate the number of students having more than 12.

6 The 1 500 students of a college are divided in the following manner :

510 are in the pre-school cycle,

30% are in the primary cycle,

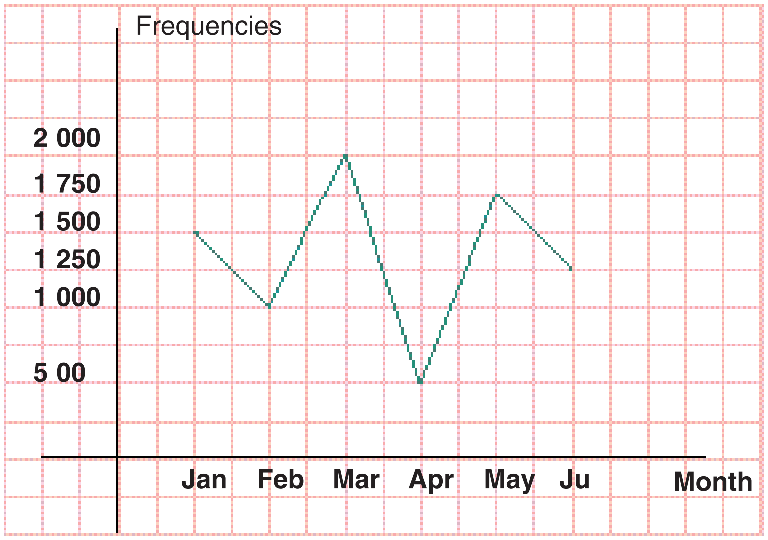
$\frac{11}{50}$ are in the elementary cycle.

1º) Complete the following table.

Cycle	Pre-school	Primary	Elementary	Secondary
Frequency	510			
Percentage		30		

2º) Represent the frequencies in percentage in a bar diagram.

7 The graph below represents the frequency polygon of cars manufactured by a factory during the first six months of the year 1997.



1° Translate these results in a frequency table and deduce the total number of manufactured cars.

2° Represent the relative frequencies in a table and draw the corresponding bar diagram.

TEST

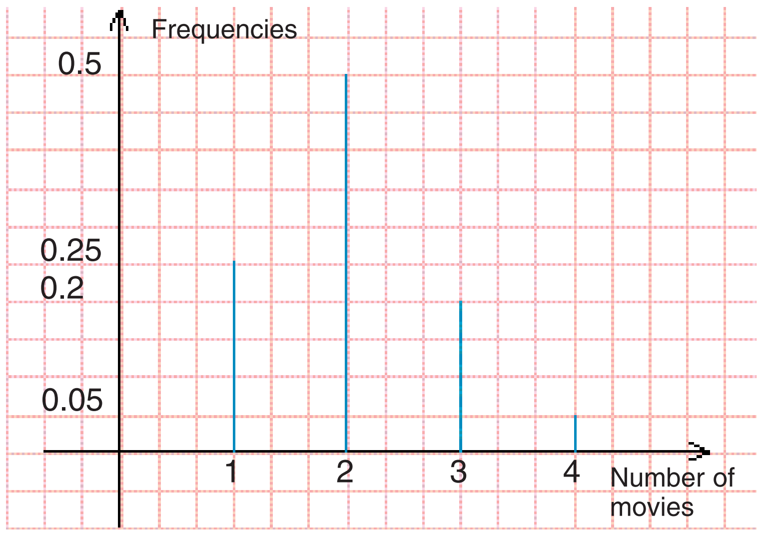
1 Answer by True or false. **(7.5 points)**

1° A dice is thrown twenty times. The number of times of appearance of each digit is shown in the following table :

a) The studied character is qualitative. **(0.5 point)**
b) The studied character is quantitative. **(0.5 point)**
c) The number of appearance of 5 is 3. **(0.5 point)**
d) The frequency of 2 is 3. **(0.5 point)**
e) The relative frequency of 6 is 0.2 . **(0.5 point)**

Digit	1	2	3	4	5	6
Frequency	2	3	5	1	5	4

2° The adjacent bar diagram represents the frequencies of movies watched by the 60 students of the grade 7 class during the month of January.



a) The population is made of the movies seen. **(1 point)**
b) The studied character is quantitative. **(1 point)**
c) The relative frequency of "2" is 0.4 . **(1 point)**
d) 25 % of the students have seen a movie. **(1 point)**
e) The frequency of "3" is 12 . **(1 point)**

2 **(12.5 points)**

"Autumn
The dawn is less clear, the air is less hot
the sky is less pure, the evening is grim and the stars are less bright"

In this poem, the number of letters of each word is studied
(for example : stars : 5 letters)

1° Complete the frequency table. **(3 points)**

Number of letters	1	2	3	4	5	6	7
Frequency (number of words)	0					3	

2° Draw the relative frequency table. **(3 points)**

3° Draw the frequency polygon. **(4 points)**

4° **a)** What is the number of words made with more than four letters ? **(1 point)**
b) What is the percentage of these words ? **(1.5 points)**

6 POWERS

Objectives

- Use the notation a^n .
- Calculate the product and the quotient of two powers of the same positive number.
- Calculate the product and the quotient of two positive numbers.
- Calculate a power of a power of a positive number.

CHAPTER PLAN

1 POWER OF A POSITIVE NUMBER

Activity

Observe and complete the following table .

$5 \times 5 =$	5^2
$4 \times 4 \times 4 =$	$4^{\cdots}$
$7 \times 7 \times 7 \times 7 =$	7^4
$8 \times 8 \times 8 \times 8 \times 8 =$	$8^{\cdots}$
.... =	3^6
.... =	10^3
$10 \times 10 \times 10 \times 10 \times 10 =$	....

Definition

- a is a strictly positive number and n is a natural number greater than 1 : $\boldsymbol{a \times a = a^2}$; we read « ***a* squared** » or « ***a* exponent 2** ».
- $\underbrace{\boldsymbol{a \times a \times a}}_{3} = \boldsymbol{a^3}$; we read « ***a* cubed** » or « ***a* exponent 3** ».
- $\underbrace{\boldsymbol{a \times a \times \ldots \times a}}_{n} = \boldsymbol{a^n}$; we read « ***a* exponent *n*** » or « ***a* to the power of *n*** ».

$\boldsymbol{a^n}$ is called the $\boldsymbol{n^{th}}$ **power of *a***.
a is called the **base** and ***n* is the exponent** of this power.
Particular powers : $\boldsymbol{a^1 = a}$ and $\boldsymbol{a^0 = 1}$.

EXAMPLES

7^3 , 7^5 , 7^2 are powers of 7 .

8^5 is called the fifth power of 8 . We read «8 exponent 5» .

2 PROPERTIES

Property 1

Activity

1º) Calculate : $2^2 = \dots$; $2^3 = \dots$

2º) Calculate : $2^2 \times 2^3 = \dots$; $2^5 = \dots$

3º) What do you deduce ?

Rule

a is a strictly positive number, m and n are two natural numbers :

$$\boxed{a^m \times a^n = a^{m+n}}$$

EXAMPLES

- $5^4 \times 5^3 = 5^{4+3} = 5^7$.
- $10^2 \times 10 = 10^{2+1} = 10^3$.
- $7^4 \times 7^3 \times 7 = 7^{4+3+1} = 7^8$.
- $a^2 \times a^3 = a^5$.

Application 1

Write each of the following products in the form of one power :

$4^5 \times 4^2$; $10^5 \times 10^3$; $(12.3)^4 \times (12.3)^5$; $3^0 \times 3^{13}$; $4^2 \times 4^5 \times 4^3$; $a \times a^2$.

Property 2

Activity

1º) Calculate : $2^2 = \dots$; $3^2 = \dots$

2º) Calculate : $2^2 \times 3^2 = \dots$; $(2 \times 3)^2 = \dots$

3º) What do you deduce ?

Rule

a and b are two strictly positive numbers, n is a natural number :

$$\boxed{(a \times b)^n = a^n \times b^n}$$

EXAMPLES

- $(3 \times 4)^5 = 3^5 \times 4^5$.
- $(2.4 \times 5.7)^8 = (2.4)^8 \times (5.7)^8$.
- $(2 \times 3 \times 5)^4 = 2^4 \times 3^4 \times 5^4$.
- $(a \times b)^3 = a^3 \times b^3$.

Application 2

Complete the following :

1°) $(5 \times 10)^4 = 5^4 \times 10^{\cdots}$

2°) $15^8 \times 13^8 = (15 \times 13)^{\cdots}$

3°) $(\ldots \times 13.2)^7 = 5^7 \times (13.2)^{\cdots}$

4°) $(\ldots \times 5)^3 = 8 \times 5^3$.

Property 3

Activity

1°) Calculate : $\left(\frac{2}{5}\right)^3 = \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} = \ldots$

2°) Calculate : $\frac{2^3}{5^3} = \frac{2 \times \ldots \times \ldots}{5 \times \ldots \times \ldots} = \ldots$

3°) What do you notice ?

Rule

a and *b* are two strictly positive numbers, *n* is a natural number :

$$\boxed{\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}}$$

EXAMPLES

1°) $\left(\frac{3}{5}\right)^4 = \frac{3^4}{5^4}$

2°) $\left(\frac{13}{17}\right)^5 = \frac{13^5}{17^5}$

Application 3

Complete the following :

1°) $\left(\frac{4}{3}\right)^5 = \frac{4^5}{\ldots}$

2°) $\left(\frac{7}{13}\right)^{\cdots} = \frac{\ldots}{13^5}$

3°) $\left(\frac{8}{9}\right)^{13} = \frac{\ldots}{9^{13}}$

4°) $\left(\frac{8}{6}\right)^5 = \frac{\ldots}{3^5}$

Property
4

Activity

1º) Calculate : $(5^2)^3 = 5^2 \times 5^2 \times 5^2 = 5^{....}$

2º) Calculate : $5^{2\times 3} = 5^{....}$

3º) What do you notice ?

Rule

a is a **strictly positive number**, *m* and *n* are two natural numbers :

$$\boxed{(a^m)^n = a^{m \times n}}$$

EXAMPLES

- $(17^5)^3 = 17^{5\times 3} = 17^{15}$
- $[(3.4)^2]^4 = (3.4)^{2\times 4} = (3.4)^{8}$.

Application 4

Complete the following :

1º) $(8^4)^5 = 8^{\cdots}$; **2º)** $[(\ldots)^3]^{\cdots} = 13^{15}$; **3º)** $(7^{\cdots})^6 = 7^{42}$;

4º) $\left[\left(\frac{5}{3}\right)^2\right]^5 = \frac{5^{10}}{\cdots}$; **5º)** $(5^3)^{\cdots} = 5^{18}$; **6º)** $(a^2)^3 = a^{\cdots}$.

3 POWERS OF 10

Numerical example

$10^2 = 10 \times 10 = 100$

$10^3 = 10 \times 10 \times 10 = 1000$

Rule

n is a natural number : $\mathbf{10^n = \underbrace{1000 \ldots 0}_{n}}$

EXAMPLE

$10^5 = 100\ 000$

Application 5

1º) Complete the following table :

10^3	**10^4**		**10^0**			
1000		**100**		**10**	**1 000 000**	**100 000 000**

2º) Complete the following :

$\left(\frac{7}{10}\right)^3 = (0.7)^{\cdots}$; $(3.7)^{\cdots} = \frac{37^{\cdots}}{10^2}$; $(\ldots)^{\cdots} = \frac{9^3}{10^3}$; $\left(\frac{1.2}{0.4}\right)^{\cdots} = 3^4$.

4 SCIENTIFIC NOTATION

A number in **scientific notation** is written in the form of : $\boldsymbol{a \times 10^p}$ where $\boldsymbol{a}$ is a **decimal** number with $\boldsymbol{1 \leq a < 10}$ and $\boldsymbol{p}$ **is an integer.**

EXAMPLES

- The scientific notation of 28400 is 2.84×10^4 .
- The scientific notation of 631.18 is 6.3118×10^2 .

Application 6

Complete the following table :

Number 3530	**36.42**		**4.52**	
Scientific notation		**2.718×10^2**		

5 ORDER OF CALCULATION

Numerical examples

1º) To calculate $5^3 \times 2$, perform first the power 5^3 then the product $5^3 \times 2$:

$5^3 \times 2 = 125 \times 2 = 250$.

We say that the **power** has the **priority** over the **multiplication**.

2º) $5^3 + 23 = 125 + 23 = 148$.

The **power** has the **priority** over the **addition**.

3º) $5^3 - 50 = 125 - 50 = 75$.

The **power** has the **priority** over the **subtraction**.

4º) $15^2 \div 5 = 225 \div 5 = 45$.

The **power** has the **priority** over the **division**.

5º) $15 \times 20 - 50 + 20 = 300 - 50 + 20 = 270$.

The **multiplication** has the **priority** over the **addition** and the **subtraction**.

6º) $28 \div 4 + 6 - 3 = 7 + 6 - 3 = 10$.

The **division** has the **priority** over the **addition** and the **subtraction**.

Remark :

The expressions of the type : $24 \div 3 \times 5$; $36 \div 6 \div 3$ and $3 \times 12 \div 6$ are not allowed. There are parentheses that are missing.

Rules

1º) In an expression without parentheses, where there are powers, products, divisions, additions and subtractions, the calculation of the **powers** has the **priority** over the rest.

The following order is followed : **powers, multiplications and divisions** (in the order of their appearance), **then additions and subtractions**.

2º) In an expression containing parentheses, the calculation **inside the parentheses has the priority over the rest.**

EXAMPLES

- Calculate $A = 2 \times 4 \times 3^2 - 24 \div 2^3 + 5^2 \times 4 \times 2$.

$$\begin{aligned} A &= 2 \times 4 \times 9 - 24 \div 8 + 25 \times 4 \times 2 \\ &= 72 - 3 + 200 \\ &= 269. \end{aligned}$$

- Calculate $B = 3 \times (2 + 5^2) - 3 \times (9 - 7)^3$.

$$\begin{aligned} B &= 3 \times (2 + 25) - 3 \times 2^3 \\ &= 3 \times 27 - 3 \times 8 \\ &= 81 - 24 \\ &= 57. \end{aligned}$$

Exercises and problems

All the used letters represent strictly positive numbers.

For testing the knowledge

1 Complete the following table.

The power	it is read	it's a power of	it's the product
4^3	4 exponent 3	4	$4 \times 4 \times 4$
10^4			
	7 exponent 5		
			$8 \times 8 \times 8 \times 8$

2 Write the following in the form of a power :

1°) The square of 4.

2°) The cube of $\frac{5}{7}$.

3°) The fifth power of 7.

4°) 8.9 exponent 9.

5°) 10 exponent 3.

6°) The power of $\frac{1}{3}$ exponent 7 .

7°) The opposite of the square of 13.2.

8°) The opposite of the fourth power of 19.

9°) The square of x .

3 **1°)** Calculate : 2^3 ; 3^3 ; 4^3 ; 5^3 ; 6^3.

2°) Find the intruder :

$\frac{1}{8}$; $\frac{1}{27}$; $\frac{1}{216}$; $\frac{1}{144}$; $\frac{1}{64}$.

4 Write in the form of a power of 10.

1000 ; 100 000 ; 1 ;
10 ; 1 000 000 000 ;
$10^2 \times 10^5$; $(10^4)^5$;
$(10^2)^3 \times (10^7)^2$; $(10^7 \times 10^5)^2$;
$10^{16} \times (10^3)^4$.

5 How many zeros are found in the writing of each of the following numbers ?

$(10^4)^2$; $10^4 \times 10^2$; $10^5 \times 10^4$;
$10^7 \times 10^0$; $(10^{10})^{10}$;
$10^{10} \times 10^{10}$; $(10^{18} \times 10^{15})^0$.

6 Write the following in scientific notation.

737 million ; 85 billion ;
71 hundred thousand ; 240 million ;
13.7 billion.

7 Complete.

1º) $b^7 = b^2 \times b^{\cdots}$ **2º)** $1\ 000\ 000 = 10^{\cdots}$

3º) $x^2 \times x^{\cdots} = x^9$ **4º)** $(a \times b)^{\cdots} = a^7 \times \ldots$

5º) $(10^2)^{\cdots} = 10^8$ **6º)** $a \times a^n \times a^{\cdots} = a^{n+3}$

7º) $\left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right)^4 \times \left(\frac{2}{3}\right)^4 = \left(\frac{2}{3}\right)^{\cdots}$

8º) $\left[\left(\frac{3}{5}\right)^2\right]^{\cdots} = \left(\frac{3}{5}\right)^8$.

8 Write the following in the form of one power :

$2^7 \times 2^5$; $\left(\frac{3}{5}\right)^8 \times \left(\frac{3}{5}\right)^9$

$9^{21} \times 9$; $(5.1)^4 \times (5.1)^{14}$

$(10.2) \times (10.2)^{12}$; $2^2 \times 4^7$

$2^3 \times 4^6$; $3^6 \times 9^{15}$

$8^2 \times 16$; $\left(\frac{25}{9}\right)^3 \times \left(\frac{5}{3}\right)^2$

$(a^3)^2 \times a$; $\left(\frac{a}{3}\right)^3 \times \left(\frac{a}{3}\right)^2$.

9 Write the following in the form of a product of two powers :

$2^5 \times 2^3 \times 3^{11} \times 3^9$

$3^9 \times 5^2 \times 3^{10} \times 5^{17}$

$(2^3)^2 \times (3^5)^3 \times 2 \times 3^2$

$(7.2)^3 \times 5^{10} \times (7.2)^{17} \times 7.2 \times 5$

$(3.5)^7 \times (7.01)^4 \times (3.5)^{13} \times (7.01)^9$

$(a^2)^5 \times (b^2)^6 \times a^5 \times b$

$(a^3 \times b^2)^5 \times (a^4 \times b^3)^2 \times a$.

10 Write each of the following expressions in the form of products of 2, 3, 5, 7 or 11.

$A = (2 \times 5)^3 \times 2^2 \times 5^3$

$B = 3^2 \times 7 \times (5 \times 3)^4 \times 7$

$C = 9^2 \times 27 \times (5^2)^3 \times 25$

$D = 9 \times 16 \times (27 \times 4)^2$

$E = 55 \times (2 \times 3^2)^2 \times 22^2$.

11 Calculate each of the following expressions.

$A = 3 - 4^2$ $B = -7 + 3^2$

$C = 5 \times 2 - 4^2$ $D = 2 \times 3^2 - 7 \times 2$

$E = 3^2 \times 5 - 5 \times 2^3$ $F = -5^2 - 7^2$

$G = 14^0 \times 3 - 6 \times 3^3$ $H = -7^2 + 3^2 \times (-2)$

$I = 3^2 \times 2^3 - (2 \times 5)^2$.

12 Perform.

$A = (13 - 2 \times 5)^2 + 16$

$B = (26 - 2^2 \times 5)^2 - 2 \times 3$

$C = 3^2 - (-5 + 2 \times 3)^2$

$D = 5^2 \times 2 - 3 \times (5^2 - 3 \times 8)^{100}$

$E = 5 \times 10^4 + 4 \times 10^3 + 2 \times 10^2$

$F = (9 - 2 \times 4)^5 - 7 \times 2^2 + (5 - 3)^2$.

13 Calculate in the most rapid way.

$A = 4^5 \times (0.25)^5$ $B = 5^{10} \times (0.2)^{10}$

$C = 50^{13} \times (0.2)^{13}$ $D = 40^5 \times (0.025)^5$

$E = 4^{11} \times (0.25)^{11} + 14 \times 8^{11} \times (0.125)^{11}$

$F = 2^5 \times 5^4$ $G = 8^{13} \times (0.125)^{12}$

$H = 9 \times 5^{12} \times (0.2)^{11} + 100^{13} \times (0.01)^{13}$.

14 Answer by true or false.

1°) The square of three is equal to the cube of two.

2°) $4^2 = 2^4$.

3°) $(0.5)^2 = 0.25$.

4°) $(0.3)^2 = 0.9$.

5°) 10^9 is a ten-digit number.

6°) 10^{13} is a thirteen-digit number.

7°) $3^4 \times 3^5 = 3^{20}$.

8°) $13^4 + 13^5 = 13^9$.

9°) $3^3 + 3^3 + 3^3 = 3^4$.

10°) $\dfrac{6^4}{3^4} = \dfrac{6}{3}$.

11°) $\dfrac{14^3}{7^3} = 2^3$.

12°) $(10^5)^2$ is different from 10^{25}.

13°) $4^3 \times 5^3 = 20^3$.

14°) $(12^2)^5 = 12^{10}$.

15°) $6^2 \times 3^4 = 18^6$.

16°) $10^3 + 10^3 = 20^3$.

15 Complete .

1°) $\left(\dfrac{2}{3}\right)^4 \times \left(\dfrac{2}{3}\right)^5 = \left(\dfrac{2}{3}\right)^{\cdots} = \dfrac{2^{\cdots}}{3^{\cdots}}$

2°) $\left(\dfrac{2^4}{7}\right)^5 = \dfrac{2^{\cdots}}{7^{\cdots}}$

3°) $\left(\dfrac{2}{7^3}\right)^5 = \dfrac{2^{\cdots}}{7^{\cdots}}$

4°) $\left(\dfrac{3^4}{8^3}\right)^5 = \dfrac{3^{\cdots}}{8^{\cdots}}$

5°) $\left(\dfrac{3^2}{5}\right)^4 = \dfrac{9^{\cdots}}{5^{\cdots}}$

6°) $\left(\dfrac{3^2}{8}\right)^5 = \left(\dfrac{3^2}{2^{\cdots}}\right)^5 = \dfrac{3^{\cdots}}{2^{\cdots}}$

7°) $\left(\dfrac{5}{16}\right)^4 = \dfrac{5^{\cdots}}{2^{\cdots}}$

8°) $\left(\dfrac{125}{32}\right)^6 = \dfrac{5^{\cdots}}{2^{\cdots}}$.

16 Complete .

1°) $\dfrac{27}{8} = \left(\dfrac{\cdots}{\cdots}\right)^3$. **3°)** $(0.5)^4 = \dfrac{1}{2^{\cdots}}$.

2°) $\dfrac{1}{32} = \left(\dfrac{\cdots}{\cdots}\right)^5$. **4°)** $(0.4)^3 = \left(\dfrac{\cdots}{\cdots}\right)^3$.

For seeking

17 Fill the box with the correct answer.

1°) $4 \times 10^5 + 3 \times 10^4 + 2 \times 10^2 + 3 \times 10^0 = \boxed{}$ 4323; 430203; 43023

2°) $-8^2 = \boxed{}$ -16; 64; -64

3°) $-1^{21} = \boxed{}$ -21; 1; -1

4°) $3^2 + 4^2 = \boxed{}$ 7^2; 14^2; 5^2

5°) $-2^2 + 2^2 = \boxed{}$ 0; 8; -8

6°) $2 \times 5^2 = \boxed{}$ 10^2; 50; 20.

18 Complete according to the given example.

$300 \times 12\,000 = 3 \times 10^2 \times 12 \times 10^3 = 36 \times 10^5$.

$4\,000 \times 1\,100 = \ldots$; $140 \times 6\,000 = \ldots$; $502\,000 \times 70 = \ldots$; $100 \times 10\,000 = \ldots$.

19 Write in the form of a product of three powers.

1º) $(2 \times a^2 \times b^3)^4 \times 2a^2$

2º) $(3 \times a^3 \times b^2)^3 \times (3^2 \times a^5 \times b)^3$

3º) $5^2 \times 25 \times 3^2 \times 7 \times 35$

4º) $12^2 \times 18^3 \times 25^4$.

20 The following are the respective distances from the major planets to the sun :

Jupiter : 7792×10^5 km

Pluto : 57×10^9 km

Mars : 228 million km

Saturn : 1.4 billion km

Mercury : 59.14 million km

Earth : 150 million km

Neptune : 4 500 000 000 km

Uranus : 2.87 billion km.

Write in scientific notation each of the above distances.

21 The human blood contains, on an average, five million of red blood cells per mm^3. What is the total number of red blood cells in five liters of blood ? (1 liter = 1 dm^3).

22 The physicist Avogadro proved that 18gm of water contain approximately 6.03×10^{23} water molecules.
Calculate the number of water molecules contained in 18 kg.

23 Write in scientific notation.

1º) $(0.3)^2 \times 40^3$

2º) $\left(\frac{2}{5}\right)^2 \times 60^2$

3º) $0.027 \times 5^3 \times 4^2 \times 6$

4º) $(0.02)^3 \times (40)^6$

5º) $(0.8)^2 \times (60)^3$

6º) $\left(\frac{2}{5}\right)^2 \times (30)^3$.

24 Write in the form of one power the following :

1º) $\left(\frac{3^2}{7}\right)^2 \times \left(\frac{9}{7}\right)^3$ **2º)** $\left(\frac{3^3}{8}\right)^2 \times \left(\frac{9}{2^2}\right)^3$

3º) $\left(\frac{2^3 \times 3^3}{5^3}\right)^4$ **4º)** $(0.2)^3 \times \left(\frac{1}{5^2}\right)^2$.

25 Cross numbers (You may use the calculator).

Horizontally

1) 6^2 ; $10^4 - 1$

2) fifth power of 3

3) $3^4 \times 2^8$

4) 5^4 ; $5^4 - 10^2$

5) 8×10^5

6) 5×2^3

7) $10^6 + 8 \times 10^5 + 10^3 + 10^2 + 1$

Vertically

1) 2^{15}

2) $2^2 \times 4^2$; 2^{11}

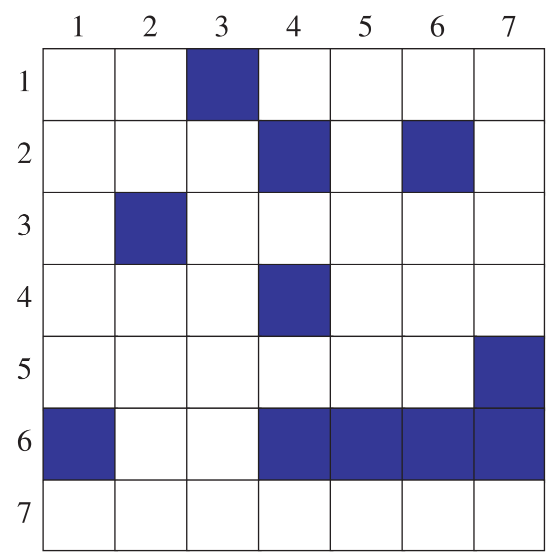
3) $13 \times 5^5 \times 2^3$

4) ...

5) $5^6 \times 6$

6) 5×2^6

7) $5^5 \times 3 - 10$.



TEST

1 Answer by true or false. **(3 points)**

1º) $7^2 + 8^2 \neq 15^2$. **2º)** 10^{10} is an 11-digit number. **3º)** $10^1 = 1^{10}$.

4º) $(10^5)^2 = 10^{25}$. **5º)** $x \times x = x^2$.

2 Write each of the following expressions in the form of a product of powers of 2, 3, 5 or 7. **(8 points)**

1º) $2^4 \times 4^2 \times 25 \times 7 \times 21 = ...$ **2º)** $125 \times 27 \times 35 \times 36 = ...$

3º) $45 \times 25 \times 36 \times 35 \times 49 \times 7 = ...$ **4º)** $21 \times 35 \times 100 \times 84 = ...$

3 Complete : **(3 points)**

1º) $\left(\frac{49}{8}\right)^{\cdots} = \frac{\cdots}{2^6}$; **2º)** $\frac{10}{(21)^2} = \frac{10}{3^2 \times 7^{\cdots}}$; **3º)** $\left[(3.2)^2\right]^{\cdots} = \frac{2^{30}}{10^{\cdots}}$.

4 Write each of the following numbers in scientific notation :

1998 ; 2731.425 ; 134.05×10^4 ; $(0.5)^3 \times (800)^2 \times 6$.

(2 points)

5 Perform .

$A = 3^2 - 5 \times (3 - 7) - 2^3 \times (1.7 + 2.5 + 150.75)^0$.

$B = (19 - 3 \times 6)^{12} - 13 \times 2^2 + 15 - 2 \times 3^3$. **(4 points)**

7 PRIME NUMBERS

Objectives

- Recognize a prime number.
- Recognize whether a number is prime or not.
- Apply Eratosthenes' method to calculate all the prime numbers less than 100.
- Know and use the algorithm of successive divisions.

CHAPTER PLAN

COURSE

EXERCISES AND PROBLEMS

TEST

1 PRIME NUMBERS

Activity

1º) Given the natural number 13.

a) Are 1 and 13 divisors of 13?

b) Does 13 have other divisors ?

c) What is the number of the divisors of 13?

d) Does 13 admit only two divisors ?

2º) **a)** List the divisors of 12.

b) What is the number of the divisors of 12?

c) Does 12 admit only two divisors ?

Definition

Let p be a natural number such that $p \geq 2$.

p is said to be **prime** if it admits **only two divisors : 1 and p**.

Remark :

0 and 1 are not prime.

EXAMPLES

- 3 admits only two divisors : 1 and 3; 3 is therefore prime.
- 13 admits only two divisors : 1 and 13 ; 13 is therefore prime.
- 12 admits more than two divisors ; hence 12 is not prime.

Application 1

1º) List the divisors of 20. Is 20 prime ? Justify.

2º) State whether each of the following numbers is prime or not :

11 ; 15 ; 17 ; 24 ; 29.

2 PRIME NUMBERS LESS THAN 100

Activity

The goal of this activity is to find the prime numbers that are less than 100.
Below is the list of the natural numbers from 0 till 100.

0	1	2	3	4	5	6	7	8	9	10
	11	12	13	14	15	16	17	18	19	20
	21	22	23	24	25	26	27	28	29	30
	31	32	33	34	35	36	37	38	39	40
	41	42	43	44	45	46	47	48	49	50
	51	52	53	54	55	56	57	58	59	60
	61	62	63	64	65	66	67	68	69	70
	71	72	73	74	75	76	77	78	79	80
	81	82	83	84	85	86	87	88	89	90
	91	92	93	94	95	96	97	98	99	100

1º) Cross out the numbers 0 and 1 (0 and 1 are not prime).

2º) 2 is prime, so cross the multiples of 2 , except 2.

3º) 3 is prime, so cross the multiples of 3 , except 3.

4º) 5 is prime, so cross the multiples of 5 , except 5.

5º) 7 is prime, so cross the multiples of 7 , except 7.

Result

The numbers that are not crossed out are the prime numbers less than 100.
This method is known as Eratosthenes' method.

3 RECOGNIZE PRIME NUMBERS

Activity

1º) Complete the following table.

The number	131							
The divisor	2	3	5	7	11	13		
The quotient	65	43	26					
The remainder	1	2						

What is the quotient of 131 by 13 ? Compare this quotient to 13.

2º) Complete the following table (stop once the obtained remainder is zero).

The number	187								
The divisor	2	3	5	7	11	13	17		
The quotient	93								
The remainder	1								

Is 187 prime ? justify.

Rule

To recognize whether a number is prime, we divide it successively by the prime numbers : 2, 3, 5, 7, … until we obtain :

- no remainder, hence the number is not prime,

- a quotient which is less or equal to the divisor with a non-zero remainder. The number is therefore prime.

Application 2

State whether the following numbers are prime : 221 ; 367 ; 231.

Remark :

2 is the only even prime number.

Exercises and problems

For testing the knowledge

1

1º) Is 48 divisible by 2 ? Justify .
Is 48 prime ? Why ?

2º) Is 309 divisible by 3 ? Justify .
Is 309 prime ? Why ?

3º) Is 728 divisible by 4 ? Justify .
Is 728 prime ? Why ?

4º) Is 275 divisible by 5 ? Justify .
Is 275 prime ? Why ?

5º) Is 927 divisible by 9 ? Justify .
Is 927 prime ? Why ?

6º) Is 1210 divisible by 10 ? Justify .
Is 1210 prime ? Why ?

2

1º) Complete.

The natural number	Its divisors	The number of its divisors	Prime or not
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			

2º) List the prime numbers that are less than 14.

3 Tell whether each number is prime or not :

7 ; 16 ; 23 ; 27 ; 29 ; 31 ; 100.

4 List the first ten prime numbers.

5 Give three divisors of 4×13 ; is 52 prime? Justify.

6 **1º)** Is11 prime ?

2º) Is 17 prime ?

3º) Is $(17 - 11)$ prime ?

4º) Is $(11 + 17)$ prime ?

7 Answer by true or false.

1º) 31 is prime.

2º) Every odd number is prime.

3º) 1 is an odd prime number.

4º) 129 is not prime.

5º) Any prime number other than 2 is odd.

6º) 2 is the only even prime number.

7º) 0 is not prime.

8º) Any even number is not prime.

9º) Any even number other than 2 is not prime.

For seeking

8 **1º)** List the divisors of 12.

2º) What is the least divisor of 12 other than 1?

3º) Is this divisor prime? Justify.

9 **1º)** List the divisors of 45 .

2º) What is the least divisor other than 1? Is this divisor prime? Justify.

10 **1º)** List the divisors of 30.

2º) Does 30 admit a prime divisor ? Which one ?

3º) Does 30 admit other prime divisors ? Which ones ?

11 Justify why each of the following numbers is not prime.

951 ; 10 101 ; 234 ; 13×17 ; 7 325 ; $5 \times 7 \times 12$.

12 Are 437 and 491 prime ?

13 Find two prime numbers knowing that their sum is 50. List all the possibilities.

TEST

1 Are the following numbers prime ? Justify.

1 ; 5 ; 6 ; 13 ; 23 ; 27 ; 39 ; 41 . **(4 points)**

2 Justify why each of the following numbers is not prime.

10 011; (19×23); 7 171; 4 444. **(2 points)**

3 State whether each number is prime or not.

1º) 2 **2º)** 19 **3º)** (19 + 2) **4º)** (19 – 2)

5º) 3 **6º)** 17 **7º)** (17 + 3) **8º)** (17 – 3) . **(4 points)**

4 **1º)** What is the greatest prime number less than 26? **(1 point)**

2º) What is the least prime number greater than 24? **(1 point)**

5 Are 253 and 257 prime ? **(3 point)**

6 **1º)** Show that the sum of the three consecutive numbers :

16 ; 17 and 18 is not a prime number. **(1 point)**

2º) Do the same with 29 ; 30 and 31 **(1/2 point)**

7 Find two prime numbers having a sum of 30.

Are there many possibilities ? **(2 points)**

8 Give three examples of two prime numbers having their sum also prime. **(1.5 point)**

8 DECOMPOSITION OF A NATURAL NUMBER INTO A PRODUCT OF PRIME FACTORS

Objective

Know how to decompose a number into prime factors.

CHAPTER PLAN

1 DECOMPOSITION OF A NATURAL NUMBER INTO A PRODUCT OF PRIME FACTORS

Activity

Given the number 30.

1º) What is the least divisor of 30 other than 1?

2º) Complete : $30 = 2 \times \ldots$

3º) Is 15 prime?

4º) What is the least divisor of 15 other than 1?

5º) Complete : $15 = 3 \times \ldots$

6º) Is 5 prime ?
«$\mathbf{2 \times 3 \times 5}$» is the only decomposition of 30 into a product of prime factors.

Property

Any non-zero natural number **can be written** as a **product of prime factors,** and this prime factorization is **unique**.

EXAMPLE

$42 = 2 \times 3 \times 7$

2 PRACTICAL METHOD OF DECOMPOSITION

Write 180 as a product of prime factors.

Horizontal method

- 2 is the smallest divisor of 180 other than 1 : $180 = 2 \times \mathbf{90}$
- 2 is the smallest divisor of 90 other than 1 : $90 = 2 \times \mathbf{45}$
- 3 is the smallest divisor of 45 other than 1 : $45 = 3 \times \mathbf{15}$
- 3 is the smallest divisor of 15 other than 1 : $15 = 3 \times \mathbf{5}$

Hence : $180 = 2 \times 90$

$180 = 2 \times 2 \times 45$

$180 = 2 \times 2 \times 3 \times 15$

$180 = 2 \times 2 \times 3 \times 3 \times 5$

$180 = 2^2 \times 3^2 \times 5$

Vertical method

$$\begin{array}{r|l} 180 & \mathbf{2} \\ 90 & \mathbf{2} \\ 45 & \mathbf{3} \\ 15 & \mathbf{3} \\ 5 & \mathbf{5} \\ 1 & \end{array}$$

$180 = 2^2 \times 3^2 \times 5$

Application

Write the number 1188 as a product of prime factors.

EXERCISES AND PROBLEMS

For testing the knowledge

1 Write the following numbers as a product of prime factors 420 , 860 and 3600 .

2 Decompose in a product of prime factors each of the following numbers :

96 ; 96^2 ; 96^3 .

3 a and b are two natural numbers such that :

$a = 2^3 \times 3 \times 5^2$ and $b = 2^4 \times 3 \times 5$

Is a a divisor of b? Justify.

4 Decompose in a product of prime factors :

450 ; 70 ; 450×70 ; 450×70^2

48 ; 48^2 ; 48^3 ; $4^2 \times 12^3$; 10^2

$15^2 \times 77^3$; $8^4 \times 21^3 \times 33^3$.

5 Determine x and y so that the number 72 may be written in the form of :

$72 = 2^x \times 3^y$

6 Verify that each prime factor of 24 is also a factor of 840, but where its exponent is less than the exponent in the decomposition of 840.

For seeking

7 Decompose 144 in a product of prime factors.

Deduce that 144 is the square of a number to be determined.

8 Decompose 30 and 900 in a product of prime factors.

What do you notice about the exponents of the prime factors ?

Deduce that 900 is the square of 30.

9 Which number has for square :

$3^2 \times 5^2$; $2^4 \times 3^2 \times 5^2$; $2^6 \times 7^2 \times 4^4$

10 Decompose 1 728 in a product of prime factors.

Deduce that 1 728 is the cube of a number to be determined.

11 Write n and t in the form of a product of prime factors :

1º) $n^2 = 2^6 \times 3^4 \times 5^2$ and $t^3 = 2^9 \times 3^{12} \times 7^{21}$.

2º) $n^2 = 2^3 \times 3^5 \times 6^3$ and $t^3 = 3^{12} \times 7^3$

3º) $n^2 = 2^6 \times 3^4 \times 5^2$ and $t^3 = 2^9 \times 10^6$

4º) $n^2 = 5^4 \times 10^6$ and $t^3 = 3^6 \times 20^9$.

12 Are the numbers below the divisors of $2^3 \times 3^7 \times 5$?

1º) $2^3 \times 3^5$ **3º)** $2^4 \times 3^4$

2º) $2^3 \times 3 \times 5$ **4º)** $2^3 \times 5 \times 7$.

TEST

1 Write as a product of prime factors each of the following numbers :

15 147 ; 36×210 ; 420^3 . **(5 points)**

2 Write 7 056 as a product of prime factors.

Deduce that 7 056 is the square of a number to be determined. **(4 points)**

3 Let $a = 2^2 \times 3^2 \times 5$ and $b = 2^3 \times 3^2 \times 5 \times 7$ **(3 points)**

Is a a divisor of b? Justify.

4 Let $x = 420$ and $y = 1\ 050$

Write x and y in the form of a product of prime factors.

Deduce the prime factorization of :

$x \times y$; x^2 ; y^3. **(5points)**

5 Simplify.

(The result should be a product of prime factors)

1º) $2^3 \times 3^2 \times 7 \times 2^2 \times 3$

2º) $3^5 \times 5^2 \times 7^2 \times 3^2 \times 5^2 \times 13$

(3 points)

9 GREATEST COMMON DIVISOR AND LEAST COMMON MULTIPLE OF TWO NATURAL NUMBERS

Objective

Perform the algorithms of the calculation of the G.C.D and L.C.M of two natural numbers.

CHAPTER PLAN

COURSE

1 - Finding the greatest common divisor (G.C.D) of two natural numbers

2 - Other methods for determining the G.C.D of two natural numbers

3 - Finding the least common multiple (L.C.M) of two natural numbers

EXERCISES AND PROBLEMS

TEST

1 GREATEST COMMON DIVISOR OF TWO NATURAL NUMBERS

Activity

1º) Write the divisors of 60.

2º) Write the divisors of 84.

3º) What is the greatest common divisor of 60 and 84 ?

4º) Find the prime factorization of 60.

5º) Find the prime factorization of 84.

6º) What are the common prime factors of 60 and 84?

7º) Calculate : $2^2 \times 3$; compare this result to that of 3º).

Finding the greatest common divisor (G.C.D) of two natural numbers

Find the G.C.D of 24 and 36.

1st method

The divisors of 24 are : 1 , 2 , 3, 4 , 6 , 8 , **12** , 24.

The divisors of 36 are : 1 , 2 , 3 , 4 , 6 , 9 , **12** , 18 , 36.

The greatest common divisor of 24 and 36 is therefore **12** .

2nd method

The prime factorizations of 24 and 36 are :
$24 = 2^3 \times 3$ and $36 = 2^2 \times 3^2$.

The common prime factors of 24 and 36 are 2 , and 3,
so $2^2 \times 3 = \mathbf{12}$ is the G.C.D of 24 and 36.

We write : **G.C.D (24 , 36) = 12** .

Rule

The G.C.D of two natural numbers *a* and *b* is the product of their common prime factors found in their prime factorization, taken with their least exponent.

If the G.C.D of two natural numbers *a* and *b* is 1, then *a* and *b* are said to be relatively prime.

Application 1

1º) Determine the G.C.D of 32 and 48.
2º) Show that 15 and 28 are relatively prime.

2 OTHER METHODS FOR DETERMINING THE G.C.D OF TWO NATURAL NUMBERS

Determine the G.C.D of 110 and 45.

1st method

For $a > b$, G.C.D. (a , b) = G.C.D $(b, a - b)$

$$\begin{aligned} \text{G.C.D } (110 , 45) &= \text{G.C.D } (45, 110 - 45) \\ &= \text{G.C.D } (45, 65) \\ &= \text{G.C.D } (45, 65 - 45) \\ &= \text{G.C.D } (45, 20) \\ &= \text{G.C.D } (20, 45 - 20) \\ &= \text{G.C.D } (20, 25) \\ &= \text{G.C.D } (20, 25 - 20) \\ &= \text{G.C.D } (20, 5) \\ &= \text{G.C.D } (5, 20 - 5) \\ &= \text{G.C.D } (5, 15) \\ &= \text{G.C.D } (5, 15 - 5) \\ &= \text{G.C.D } (5, 10) \\ &= \text{G.C.D } (5, 10 - 5) \\ &= \text{G.C.D } (5, 5) \\ &= \mathbf{5} . \end{aligned}$$

G.C.D (110 , 45) = 5 .
This method is known under the name of difference.

Application 2

Calculate, using the method above, the G.C.D of 84 and 62.

2nd method

$$\begin{array}{r|r} & 2 \\ \hline 45 & 110 \\ & -90 \\ \hline & 20 \end{array} \qquad \begin{array}{r|r} & 2 \\ \hline 20 & 45 \\ & -40 \\ \hline & \mathbf{5} \end{array} \qquad \begin{array}{r|r} & 4 \\ \hline 5 & 20 \\ & -20 \\ \hline & 00 \end{array}$$

By this method, known under the name of **successive divisions** or **Euclidean Algorithm**, the G.C.D is the last non-zero remainder.
G.C.D (110 , 45) = 5 .

Application 3

Use the Euclidean Algorithm to determine the G.C.D of 48 and 76.

3 LEAST COMMON MULTIPLE OF TWO NATURAL NUMBERS

Activity

1º) Write the first seven non-zero multiples of 8.
2º) Write the first seven non-zero multiples of 6.
3º) What is the least non-zero common multiple of 8 and 6?
4º) Write the prime factorization of 8.
5º) Write the prime factorization of 6.
6º) List the prime factors that appear in the factorization of 8 or of 6.
7º) Calculate : $2^3 \times 3$; compare this result to that of 3º).

Finding the least common multiple (L.C.M) of two natural numbers

1º) Find the L.C.M of 120 and 36.
$120 = 2^3 \times 3 \times 5$ and $36 = 2^2 \times 3^2$.
L.C.M $(120 , 36) = 2^3 \times 3^2 \times 5$
$= 360.$
2º) Find the L.C.M of 45 and 105.
$45 = 3^2 \times 5$ and $105 = 3 \times 5 \times 7$.
L.C.M $(45 , 105) = 3^2 \times 5 \times 7$
$= 315.$
3º) Find the L.C.M of 8 and 15.
$8 = 2^3$ and $15 = 3 \times 5$.
L.C.M $(8 , 15) = 2^3 \times 3 \times 5$
$= 120.$

Rule

The L.C.M. of two natural numbers *a* and *b* is the product of all the prime factors of *a* and *b*, each with the highest exponent.

Application 4

Let m be the L.C.M and d be the G.C.D of 70 and 84 .

1º) Determine m and d.

2º) Verify that : $70 \times 84 = m \times d$.

Remarks :

1º) If a and b are relatively prime, then :

G.C.D $(a , b) = 1$ and L.C.M $(a , b) = a \times b$.

EXAMPLE

2 and 15 are relatively prime.

G.C.D (2 , 15) = 1 and L.C.M (2 , 15) = $2 \times 15 = 30$.

2º) If a is a multiple of b, then :

G.C.D $(a , b) = b$ and L.C.M $(a , b) = a$.

EXAMPLE

18 is a multiple of 6.

G.C.D (18 , 6) = 6 and L.C.M (18 , 6) = 18.

Application 5

Determine the L.C.M and G.C.D of :

1º) 60 and 15 **2º)** 7 and 9 **3º)** 12 and 6 **4º)** 36 and 24 **5º)** 20 and 21

Exercises and problems

For testing the knowledge

1 Determine the G.C.D of a and b in each of the following cases using the indicated method .

1º) $a = 315$ and $b = 280$ (decomposition into prime factors).

2º) $a = 630$ and $b = 375$ (Euclidean Algorithm).

3º) $a = 18$ and $b = 54$ (difference).

4º) $a = 594$ and $b = 770$ (decomposition into prime factors).

2 Determine the L.C.M of a and b in each of the following cases .

1º) $a = 75$ and $b = 120$

2º) $a = 8$ and $b = 24$

3º) $a = 12$ and $b = 49$

4º) $a = 264$ and $b = 1260$.

3 Let : $a = 240$ and $b = 360$.

1º) Write each as a product of its prime factors.

2º) Find : d = G.C.D (a , b) and m = L.C.M (a , b).

3º) Verify that : $240 \times 360 = m \times d$.

4 A carpenter has two pieces of wood : one measures 630 cm and the other 825 cm. He wants to divide them into equal parts having the longest possible length.
What will be the common length of these parts ?

5 The number of students in a school is between 350 and 400.
Find the number of students, knowing that they can be arranged into groups of 5 and of 9.

6 Determine the L.C.M and G.C.D of :

1º) 130 and 140 **2º)** 36 and 18

3º) 14 and 27 **4º)** 1260 and 132

5º) 75 and 25 **6º)** 260 and 100

7º) 320 and 504 **8º)** 360 and 1024

9º) 2670 and 2030

7 **1º)** List the divisors of each of the following numbers :
30 ; 49 ; 125 ; 19 ; 81 ; 25 ; 810 ; 250.

2º) Indicate, among the numbers above, the pairs which are formed of relatively prime numbers.

For seeking

8 A lighthouse emits two different lights : a red light every 12 seconds and a green light every 15 seconds.
Initially, these lights are emitted simultaneously. Indicate the time when they will be emitted again together.

9 We want to cover the floor of a rectangular room with equal square tiles which are the largest possible.
How many tiles are needed if the dimensions of this room are 630 cm and 462 cm ?

10 Give the prime factorization of the L.C.M and the G.C.D of the following numbers :

1º) $2^5 \times 3^2 \times 6^4 \times 7^2$ and $2^6 \times 3^4 \times 35^4 \times 13$.

2º) $2^3 \times 3^4$ and $2^4 \times 3^3 \times 5$.

3º) $3^6 \times 5 \times 7^2 \times 11^4$ and $3^6 \times 7^6 \times 11^2 \times 13$.

4º) 216 ; 540 and 756

5º) $2^8 \times 3^3 \times 5^2 \times 7$ and $2^5 \times 3^9 \times 7^2 \times 13^2$.

6º) 972 ; 1024 and 64.

TEST

1 Give the prime factorization of the G.C.D and the L.C.M of the numbers :
$a = 2^4 \times 3^3 \times 5^2 \times 11$ and $b = 2^3 \times 3 \times 5^3 \times 7$. **(2 points)**

2 Determine the G.C.D and the L.C.M of a and b in each of the following cases. **(4 points)**

1º) $a = 90$ and b = 180 **2º)** $a = 25$ and $b = 16$

3 Use the Euclidean Algorithm to determine the G.C.D of 840 and 680. **(3 points)**

4 Let : $x = 480$ and $y = 1260$.

a) Give the prime factorization of each.

b) Determine d = G.C.D (x,y) and m = L.C.M (x,y).

c) Verify that : $480 \times 1260 = m \times d$. **(4 points)**

5 Complete to determine the G.C.D of 75 and 45.
G.C.D (75,45) = G.C.D. (45,75 – 45) = … . **(2 points)**

6 Give the prime factorization of the G.C.D and the L.C.M of the three numbers :
$(12)^4$; $(2 \times 3^2 \times 5)^3$ and 270 . **(2 points)**

7 Two boats leave the same harbour. The first leaves every 8 days and the second every 12 days.
If they leave together on the 1st of May, when will they leave again together ? **(3 points)**

10 TRIANGLES REMARKABLE LINES IN A TRIANGLE

Objectives

- Know the definition of a triangle and its elements.
- Know the definition of the remarkable lines in a triangle .
- Know the definition of each particular triangle.

CHAPTER PLAN

COURSE

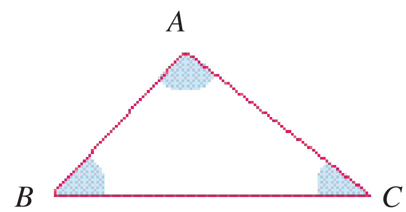
1 - Triangle
2 - Remarkable lines in a triangle
3 - Particular triangles

EXERCISES AND PROBLEMS

TEST

COURSE

1 TRIANGLE

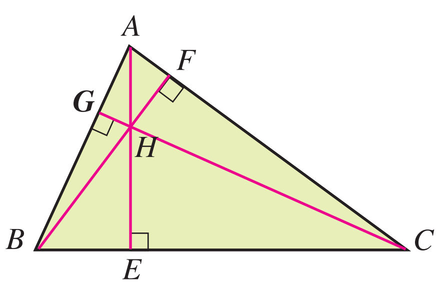


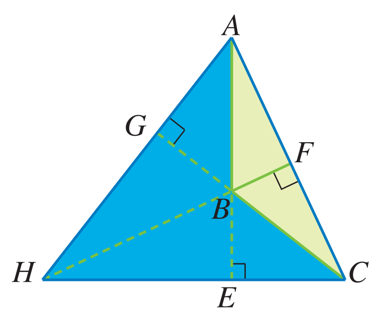
- ABC is a triangle.
- ***A***, ***B*** and ***C*** are its **vertices**.
- **[*AB*], [*AC*] and [*BC*]** are its **sides.**
- $\widehat{ABC}$, $\widehat{BCA}$ **and** $\widehat{CAB}$ are its **angles.**
- $\widehat{ABC} + \widehat{BCA} + \widehat{CAB} = 180°$**.**
- $\widehat{ABC}$ **and** $\widehat{ACB}$ are the angles adjacent to side [*BC*].
- $\widehat{ABC}$ is the angle opposite to side [*AC*] .

2 REMARKABLE LINES IN A TRIANGLE

1. Heights of a triangle

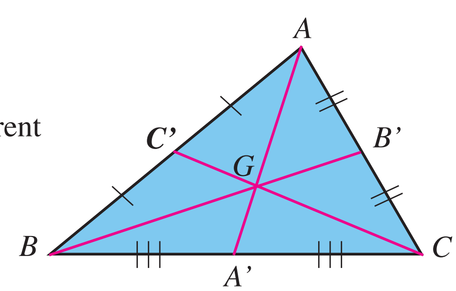
In triangle *ABC*, the three heights (or altitudes) : [*AE*], [*BF*] and [*CG*] are concurrent in point *H*, called **the orthocenter** of this triangle.





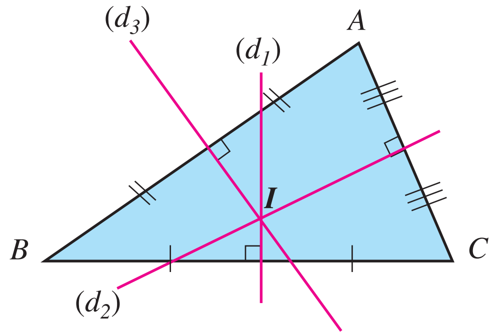
2. Medians of a triangle

In triangle *ABC*, the three medians : [*AA'*], [*BB'*] and [*CC'*] are concurrent in point ***G***, called **the center of gravity** (or centroid) of this triangle.



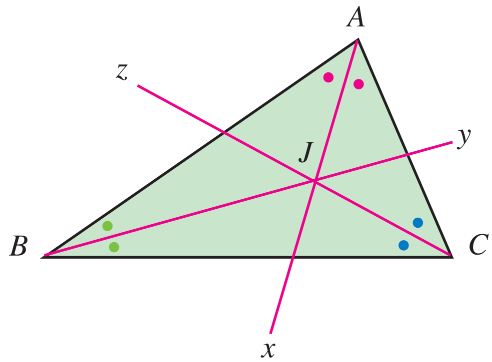
3. Perpendicular bisectors in a triangle

In triangle *ABC*, the three perpendicular bisectors (d_1), (d_2) and (d_3) meet at $\boldsymbol{I}$ which is **the center of the circle passing through the vertices** of this triangle.



4. Bisectors in a triangle

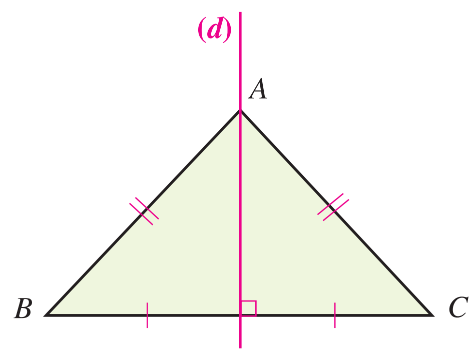
In triangle *ABC*, the three bisectors : [*Ax*), [*By*) and [*Cz*) are **concurrent in *J***, called the **incenter** of this triangle.



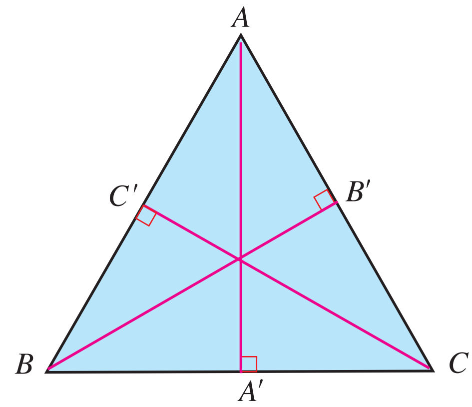
3 PARTICULAR TRIANGLES

1. Isosceles triangle

- A triangle is isosceles if it has two equal sides. For example, *ABC* is isosceles since $\boldsymbol{AB = AC}$.
- ***A*** is the **vertex** of this triangle.
- **[*BC*]**, the opposite side to the vertex, is **the base** of this triangle.
- The angles $\widehat{ABC}$ and $\widehat{ACB}$ which are **adjacent to the base** **[*BC*]** are equal.
- In triangle *ABC*, if $\widehat{ABC} = \widehat{ACB}$ **then the triangle *ABC* is isosceles.**
- The **perpendicular bisector** (*d*) of the **base [*BC*]** is **the axis of symmetry** of this triangle and **it passes through *A***.
- In an isosceles triangle, the height relative to the base, the bisector of the vertex angle, and the median relative to the base are overlapping.



2. Equilateral triangle



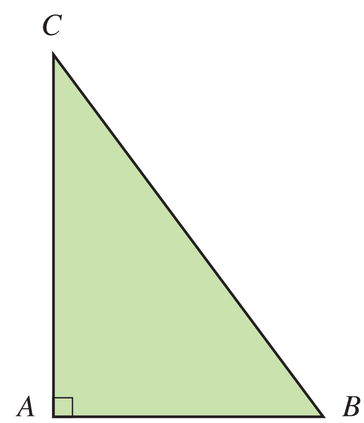
- A triangle having three equal sides is an equilateral triangle.

  For example, ABC is **equilateral since $AB = AC = BC$.**

- In an equilateral triangle, the three angles are equal **(each is 60°).**

- **The three perpendicular bisectors of the sides are the three axes of symmetry** of this triangle.

3. Right triangle



- A triangle is **right** if one of **its angles is right.**

  For example, ABC is right at A since $\widehat{BAC} = 90°$.

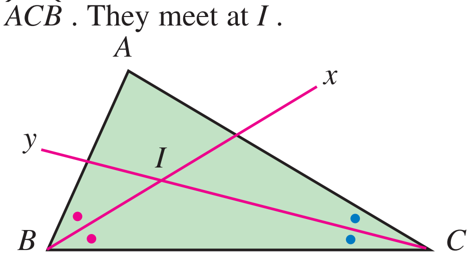
- **[BC]**, the side opposite to the right angle is called **the hypotenuse** of this triangle.

- **[AB]** and **[AC]** are **the sides of the right angle.**

Exercises and problems

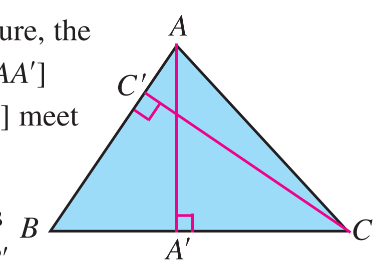
For testing the knowledge

1 In the figure below, [Bx) and [Cy) are the respective bisectors of $\widehat{ABC}$ and $\widehat{ACB}$. They meet at I .



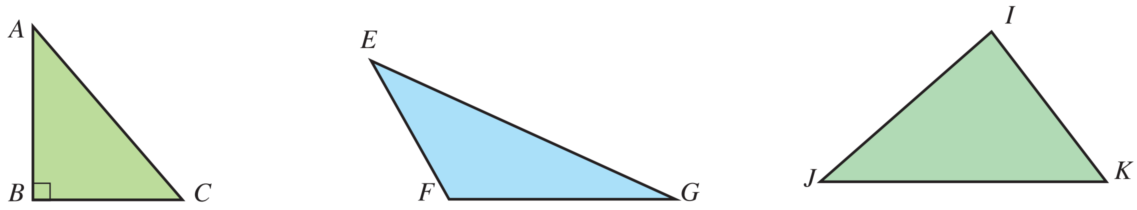
What does [AI) represent in triangle ABC?

2 In the figure, the heights [AA'] and [CC'] meet at H.

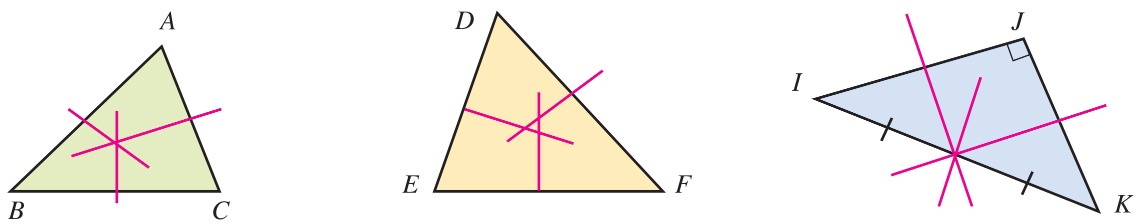


[BH) cuts (AC) at B'.
What does [BB'] represent in triangle ABC ?

3 Reproduce each of the following triangles and locate the orthocenter of each.



4 Observe the perpendicular bisectors of each triangle, then justify whether they are correct or not.



5 **1º)** Construct a right isosceles triangle ABC .

2º) Does this triangle admit an axis of symmetry ? If yes, draw it.

6 **1º)** Construct a right triangle ABC of vertex A . Let A' be the symmetric of A with respect to (BC).

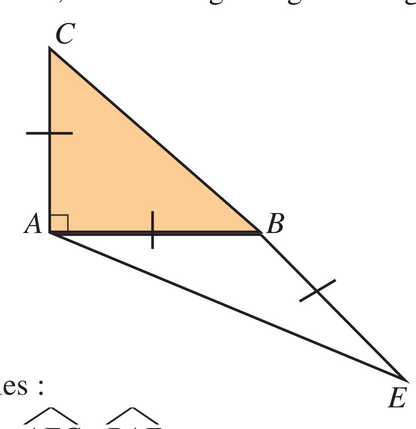
2º) Name the isosceles triangles of the figure.

3º) What does (BC) represent for segment $[AA']$?

7 **1º)** Complete : The sum of the angles of a triangle is …

2º) In a triangle ABC, let $\widehat{BAC} = 50°$ and $\widehat{CBA} = 60°$. Calculate $\widehat{BCA}$.

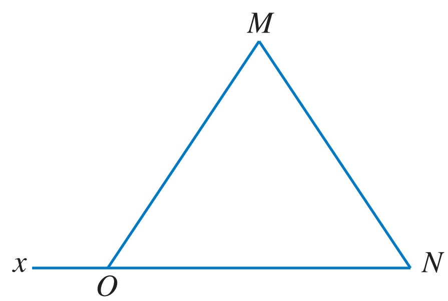
3º) In the figure below, ABC is a right-angled triangle at A . We have : $AC = AB = BE$.



Calculate each of the following angles :
$\widehat{ACB}$, $\widehat{CBA}$, $\widehat{ABE}$, $\widehat{AEC}$, $\widehat{BAE}$.

8 ABC is a right-angled triangle at A . The bisectors of $\widehat{ABC}$ and $\widehat{ACB}$ meet at I . Find the measure of $\widehat{BIC}$.

9 In the figure below, show that $\widehat{MOx} = \widehat{OMN} + \widehat{ONM}$.



10 Construct triangle ABC in each of the following cases.

1°) $BC = 4$ cm ; $\widehat{ABC} = 50°$ and $\widehat{ACB} = 60°$.

2°) $BC = 4$ cm ; $AB = 5$ cm and $AC = 6$ cm .

3°) $AB = 3$ cm ; $BC = 5$ cm and $\widehat{ABC} = 120°$.

11 Construct an equilateral triangle MIN having a perimeter of 12 cm.

12 Let ABC be a triangle such that : $BC = 75$ mm , $\widehat{ABC} = 60°$ and $\widehat{ACB} = 50°$.

The bisectors of $\widehat{BAC}$ and $\widehat{ACB}$ meet at I .

Calculate the angles of triangle AIC .

13 Let ABC be an equilateral triangle. The bisectors of its angles meet at I .

Show that $\widehat{AIB} = \widehat{AIC} = \widehat{BIC}$.

14 Construct an isosceles triangle having a side of 4 cm and a perimeter of 14 cm. (Two cases arise).

For seeking

15 Let ABC be a right-angled triangle at A . $[AH]$ is the height relative to $[BC]$.

1°) Show that $\widehat{ACH}$ and $\widehat{ABC}$ are complementary.

Show that $\widehat{BAH}$ and $\widehat{ABC}$ are complementary.

Deduce that $\widehat{ACH} = \widehat{BAH}$.

2°) Similarly show that $\widehat{ABH} = \widehat{CAH}$.

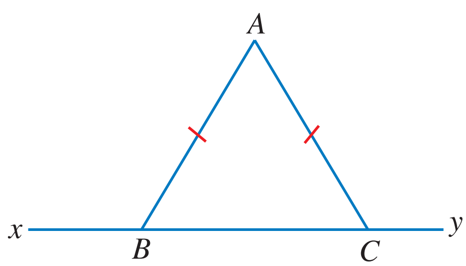
3°) Locate the orthocenter of triangle ABC .

16 Let PAL be an isosceles triangle of vertex A . The perpendicular to (PL) at P cuts (AL) at I .

1°) Show that $\widehat{API}$ and $\widehat{APL}$ are complementary as well as $\widehat{LIP}$ and $\widehat{PLI}$. Deduce that triangle IAP is isosceles .

2°) Show that :
a) A is the midpoint of $[IL]$,
b) $IL = 2PA$.

17 In the figure below, ABC is an isosceles triangle of vertex A .



1°) a) Show that $\widehat{ABx}$ and $\widehat{ABC}$ are supplementary.

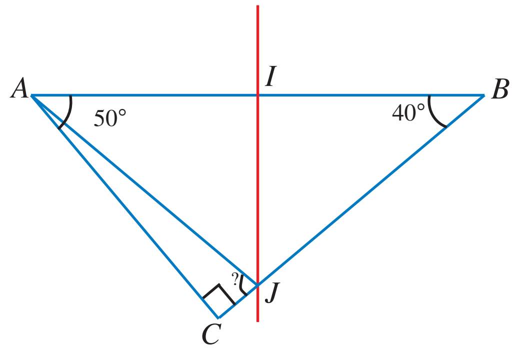
b) Is it the same for $\widehat{ACy}$ and $\widehat{ACB}$?

2°) Deduce that $\widehat{ABx} = \widehat{ACy}$.

18 Let $[Ou)$ be the bisector of any angle $\widehat{xOy}$. I is any point of $[Ou)$. The perpendicular drawn from I to $[Ox)$ cuts it at A . The perpendicular drawn from I to $[Oy)$ cuts it at B .

Show that $[IO)$ is the bisector of $\widehat{AIB}$.

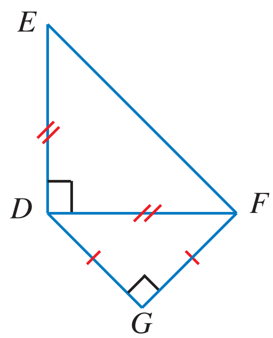
19 In the figure below, (IJ) is the perpendicular bisector of $[AB]$ and $\widehat{BAC} = 50°$.



1º) What is the nature of triangle AJB? Justify .

2º) Calculate angle $\widehat{CJA}$.

20



1º) Write the given of the coded figure above.

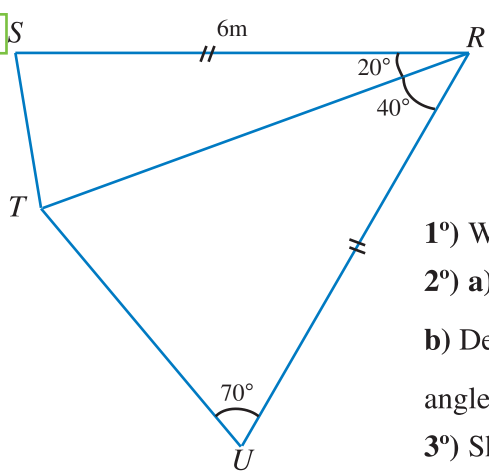
2º) a) What are the natures of triangles EDF and FGD ?

b) Show that the ray $[FD)$ is the bisector of $\widehat{EFG}$.

3º) a) Find the measure of $\widehat{EFG}$.

b) Deduce that the straight lines (EF) and (GD) are parallel.

21



1º) Write the given of the coded figure above.

2º) a) Prove that triangle RTU is isosceles .

b) Deduce that triangle RST is isosceles, then calculate angles $\widehat{RST}$ and $\widehat{RTS}$.

3º) Show that $RT = SU$.

TEST

1 **1º)** Construct triangle CAR knowing that $AC = 5$ cm , $AR = 4$ cm and $CR = 6$ cm .

2º) Construct the perpendicular bisector of $[AR]$ that cuts $[CR]$ at M .

3º) What is the nature of triangle MAR ? Justify . **(5 points)**

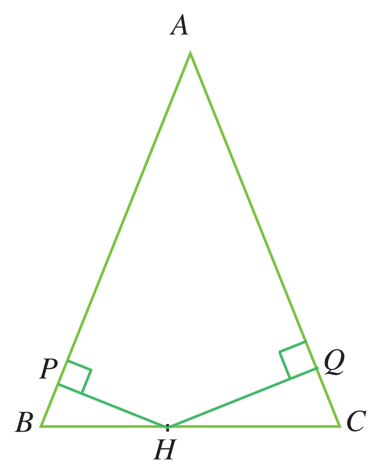
2 Let $[Ou)$ be the bisector of an angle $\widehat{xOy}$. I is any point of $[Ou)$. The perpendicular at I to $[Ou)$ cuts $[Ox)$ and $[Oy)$ at A and B .

Show that angles $\widehat{OAB}$ and $\widehat{OBA}$ are equal . **(4 points)**

3 In the adjacnt figure, ABC is an isosceles triangle of vertex A . H is any point of $[BC]$.

Show that angles $\widehat{PHB}$ and $\widehat{QHC}$ are equal .

(5 points)



4 Let ABC be a right triangle at C .

H is the midpoint of $[AB]$. The perpendicular bisector of $[AB]$ cuts (AC) and $[BC]$ at F and E respectively.

1º) Show that triangle EAB is isosceles of vertex E .

2º) a) What does F represent for triangle ABE ?

b) Deduce that (BF) is perpendicular to (AE) . **(6 points)**

11 CONGRUENT TRIANGLES (1)

Objectives

- Know the definition of two congruent triangles, as well as the corresponding parts of congruent triangles (c.p.c.t) .
- Know that if two triangles have an equal side and its two adjacent angles respectively equal, then these two triangles are congruent.

CHAPTER PLAN

COURSE

1 - Definition

2 - First case of the congruency of two triangles

EXERCISES AND PROBLEMS

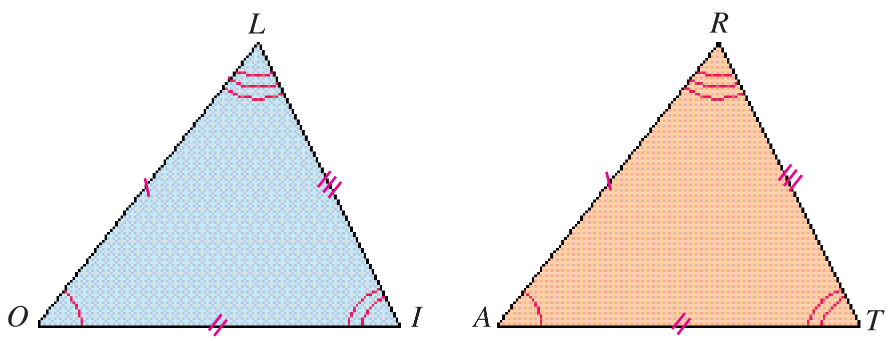
TEST

1 DEFINITION

Two **triangles** are said to be **congruent** if the three **sides** and the three angles of the first are respectively **equal** to the three sides and the three **angles** of the second.

EXAMPLE

LOI and *RAT* are congruent. They have :



$OL = AR$	$LI = RT$	$OI = AT$
$\widehat{OIL} = \widehat{ATR}$	$\widehat{LOI} = \widehat{RAT}$	$\widehat{OLI} = \widehat{ART}$

Remarks :

- [OL] and [AR] are said to be **corresponding sides** of congruent triangles. Similarly for [LI] and [RT], for [OI] and [AT].
- $\widehat{OIL}$ and $\widehat{ATR}$ are said to be **corresponding angles** of congruent triangles. Similarly for $\widehat{LOI}$ and $\widehat{RAT}$, for $\widehat{OLI}$ and $\widehat{ART}$.
- The angles facing two equal sides are equal.
- The sides facing two equal angles are equal.

2 FIRST CASE OF THE CONGRUENCY OF TWO TRIANGLES

Activity

1°) Draw $[AC] = 5$ cm. On the same side of $[AC]$, draw $\widehat{CAx} = 60°$ and $\widehat{ACy} = 40°$. $[Ax)$ and $[Cy)$ meet at L.

You have therefore constructed triangle LAC knowing the measures of one side and the two adjacent angles of this side.

2°) Do the same for drawing a triangle DEF such that $EF = 5$ cm, $\widehat{DEF} = 60°$ and $\widehat{DFE} = 40°$.

3°) On tracing paper, trace each of the two triangles LAC and DEF.

4°) Verify that these two triangles are congruent.

5°) In these two triangles, state :

1°) the equal angles . **2°)** the equal sides .

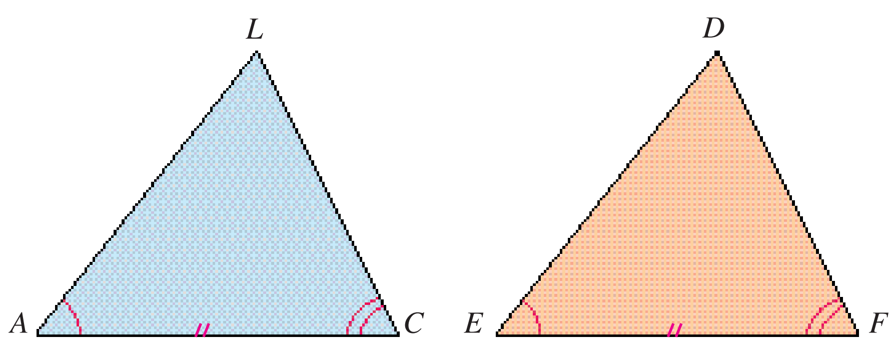
Rule

If in two triangles, a side from the first is equal to a side from the second, and the adjacent angles of these two sides are respectively equal, then the two triangles are congruent. (by a.s.a)

Example

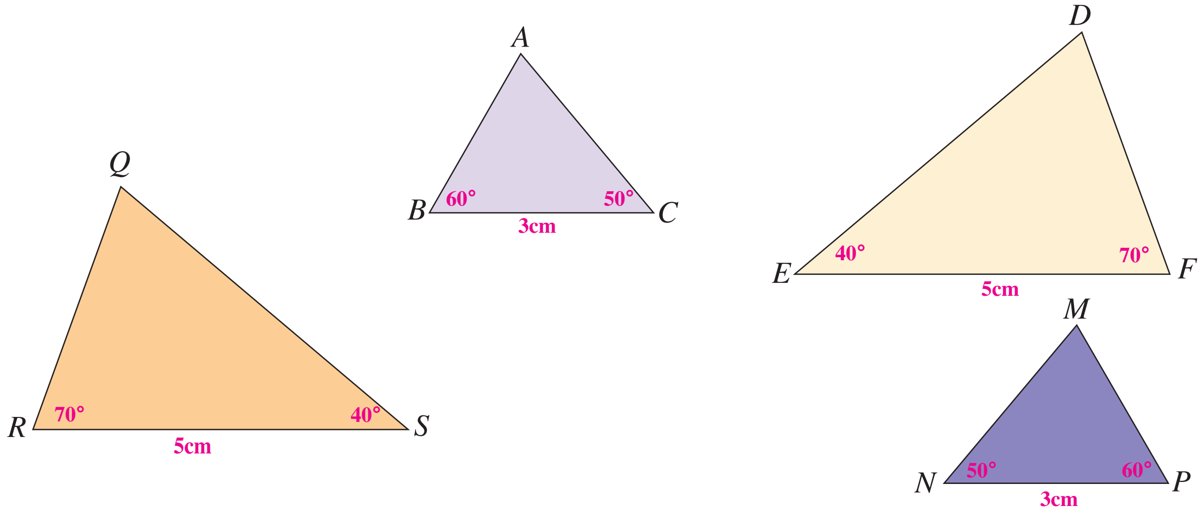
The two triangles LAC and DEF below have :

$AC = EF$, $\widehat{LAC} = \widehat{DEF}$ and $\widehat{LCA} = \widehat{DFE}$; they are therefore congruent (this has been verified in the activity).



Application

Indicate, among the given trianges, those that are equal. Justify.

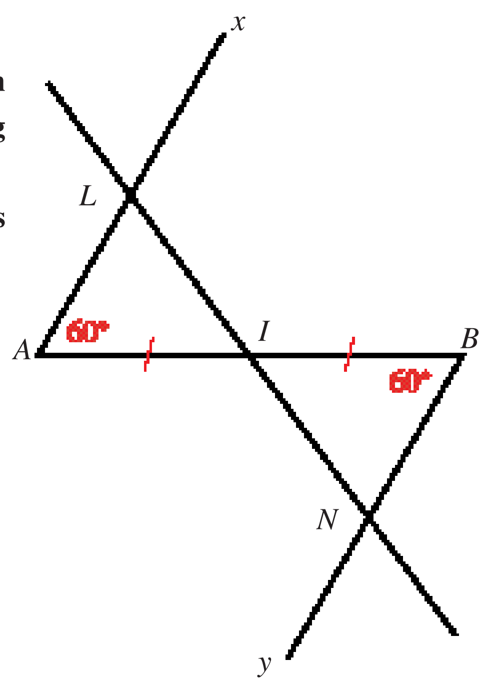


Remark :

To show that two sides or two angles are equal, we consider them as being two corresponding sides or two corresponding angles of two triangles that are proved congruent.

<u>SOLVED EXERCISE</u>

From the extremities of a segment [*AB*], draw on opposite sides of [*AB*] two rays [*Ax*) and [*By*) forming each an angle of 60° with *AB*.
From the midpoint *I* of [*AB*], draw any line that cuts [*Ax*) and [*By*) at *L* and *N* respectively.
Show that *AL* = *BN*.



Given :

$\widehat{LAB} = \widehat{NBA} = 60°$; $IA = IB$

Prove :

$AL = BN$

Proof

Consider the two triangles *LAI* and *NBI* ; they have :

$\widehat{LAI} = \widehat{NBI} = 60°$ *(given)*,
$IA = IB$ *(given)*,
$\widehat{LIA} = \widehat{NIB}$ *(vertically opposite angles)*.

These two triangles are congruent since one side and its adjacent angles from the first triangle are equal to one side and its adjacent angles from the second triangle.
All their corresponding parts are equal. In particular : *AL* = *BN*.

Exercises and problems

For testing the knowledge

1 Draw triangle *THE* in each of the following cases :

a) $ET = 63$ mm, $\widehat{ETH} = 39°$ and $\widehat{HET} = 48°$.

b) $TH = 5$ cm, $\widehat{ETH} = 45°$ and $\widehat{EHT} = 110°$.

c) $\widehat{ETH} = 32°$, $\widehat{EHT} = 48°$ and $TH = 6$ cm.

2 Let M be a point of $[Ou)$, the bisector of any angle $\widehat{xOy}$. The perpendicular drawn from M to $[Ou)$ cuts $[Ox)$ at A and $[Oy)$ at B.

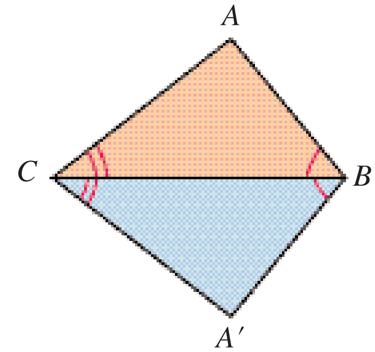
a) Show that the two triangles OMA and OMB are congruent.

b) List the corresponding parts of these two congruent triangles.

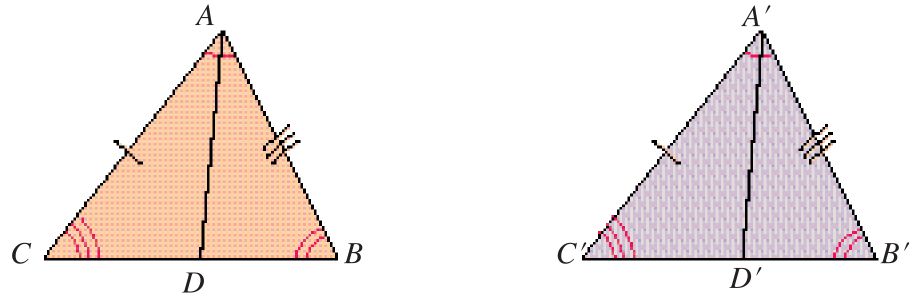
3 Using the information given in the adjacent figure ,

1º) Show that triangles ABC and $A'BC$ are congruent.

2º) Deduce then that triangles ABA' and ACA' are isosceles.



4 In the figure below, triangles ABC and $A'B'C'$ are two congruent triangles. $[AD)$ and $[A'D')$ are the bisectors of the angles $\widehat{BAC}$ and $\widehat{B'A'C'}$. Using the given, show that : $AD = A'D'$.



5 From the extremity A of a segment $[AB]$, draw on both sides of $[AB]$, two rays $[Ax)$ and $[Ay)$ such that $\widehat{BAx} = \widehat{BAy} = 50°$. Similarly from B, draw $[Bu)$ and $[Bv)$ such that $\widehat{ABu} = \widehat{ABv} = 60°$.

$[Ax)$ and $[Bu)$, which are on the same side of $[AB]$, meet at M and $[Ay)$ and $[Bv)$, which are also on the other same side of $[AB]$, meet at N.

Show that the two triangles AMB and ANB are congruent.

6 From the extremities E and F of a segment $[EF]$, and on opposite sides of this segment, draw $[Ex)$ and $[Fy)$ such that : $\widehat{FEx} = \widehat{EFy}$. The perpendicular drawn from E to $[EF]$ cuts $[Fy)$ at M . The perpendicular drawn from F to $[FE]$ cuts $[Ex)$ at N.

Show that : $EM = FN$.

7 Answer by true or false.
Two triangles ABC and $A'B'C'$ are congruent if :

1º) $AB = A'B'$ and $\widehat{BAC} = \widehat{B'A'C'}$.

2º) $AB = A'B'$ and $AC = A'C'$.

3º) $\widehat{BAC} = \widehat{B'A'C'}$, $\widehat{ABC} = \widehat{A'B'C'}$ and $\widehat{ACB} = \widehat{A'C'B'}$.

4º) $AB = A'B'$, $\widehat{BAC} = \widehat{B'A'C'}$ and $\widehat{ABC} = \widehat{A'B'C'}$.

5º) $AC = A'C'$, $\widehat{BAC} = \widehat{B'A'C'}$ and $\widehat{ACB} = \widehat{A'C'B'}$.

6º) $BC = B'C'$, $\widehat{ABC} = \widehat{A'B'C'}$ and $\widehat{ACB} = \widehat{A'C'B'}$.

For seeking

8 On the sides $[Ox)$ and $[Oy)$ of an angle $\widehat{xOy}$, place the points E and F respectively, such that : $OE = OF$.

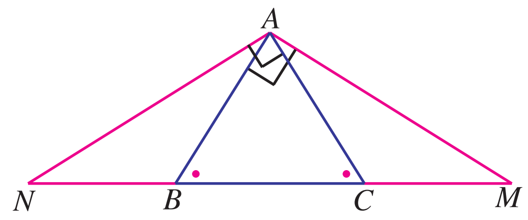
The perpendicular drawn from E to $[Ox)$ cuts $[Oy)$ at K and the perpendicular drawn from F to $[Oy)$ cuts $[Ox)$ at L.

a) Show that $OK = OL$ and $\widehat{OLF} = \widehat{OKE}$.
Deduce that $EL = FK$.

b) $[EK]$ and $[FL]$ meet at I. Show that : $EI = IF$ and $IL = IK$.

9 Let O be a point at a distance of 5 cm from a straight line (D). A is a point of (D). Elongate (OA) to a length $AC = OA$. The perpendiculars drawn from O and C to (D) cut it at H and K respectively.
a) What is the length of $[OH]$?
b) Show that $\widehat{HOA} = \widehat{KCA}$.
c) Show that $OH = CK$.

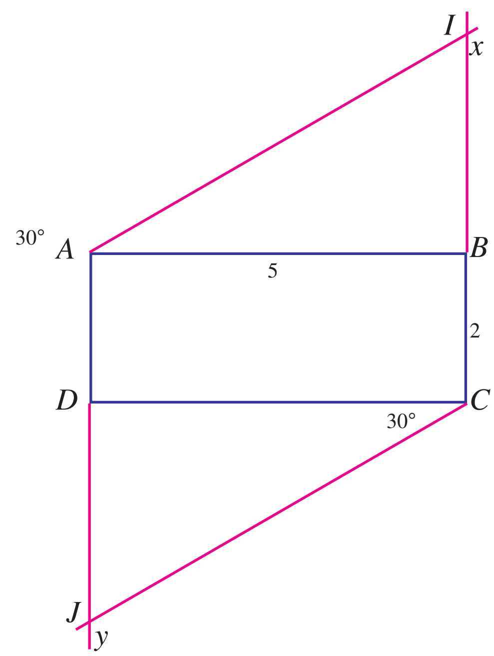
10 Let ABC be an isosceles triangle of vertex A .
The perpendicular at A to (AB) cuts (BC) at M .
The perpendicular at A to (AC) cuts (BC) at N .
1º) Show that the two triangles ANC and ABM are congruent . Deduce that $BM = CN$.
2º) Compare the two triangles ABN and ACM . Deduce that AMN is an isosceles triangle.



11 $ABCD$ is a rectangle having $AB = 5$ cm and $BC = 2$ cm .
$[Ax)$ and $[Cy)$ are two rays drawn outside the rectangle such that $\widehat{BAx} = \widehat{DCy} = 30°$.
$[Ax)$ cuts (BC) at I and $[Cy)$ cuts (AD) at J .

1º) Show that the two triangles ABI and CDJ are congruent .

2º) Deduce that $AJ = CI$.



12 Let ABC be a right-angled triangle at A such that $AB = 7$ cm and $AC = 4$ cm .
The bisector of $\widehat{BAC}$ cuts $[BC]$ at M .
Designate by I the point of $[AB]$ such that $\widehat{AMI} = \widehat{AMC}$.
1º) Show that the two triangles MAC and MAI are congruent .
2º) (MI) cuts (AC) at J .
Show that the two triangles MCJ and MBI are congruent .
3º) Deduce that triangle MBJ is isosceles.

TEST

1 Construct a triangle ABC knowing that $AB = 7$ cm, $\widehat{BAC} = 35°$ and $\widehat{ABC} = 45°$.

(2 points)

2 ABC and DEF are two triangles such that $\widehat{BAC} = \widehat{EDF}$, $\widehat{ABC} = \widehat{DEF}$ and $\widehat{ACB} = \widehat{DFE}$. Are these two triangles congruent? Why? **(2 points)**

3 Draw a triangle ABC right at A such that $AC = 5$ cm and $\widehat{BCA} = 30°$.

(2 points)

4 ABC and $A'B'C'$ are two triangles such that $\widehat{BAC} = \widehat{B'A'C'}$ and $\widehat{ABC} = \widehat{A'B'C'}$.

What is the condition that should be imposed on these two triangles so that they will be congruent? **(3 points)**

5 Given, in a triangle ABC, that $AB = AC$ and $\widehat{ABC} = \widehat{ACB}$. The bisector of $\widehat{ABC}$ cuts $[AC]$ at I and the bisector of $\widehat{ACB}$ cuts $[AB]$ at J.

Show that $BI = CJ$. **(4 points)**

6 Let O be the midpoint of a segment $[AB]$. (xy) and (uv) are the perpendiculars to (AB) passing through A and B respectively. A line passing through O cuts (xy) at C and (uv) at D.

1°) Show that $OC = OD$. **(3 points)**

2°) The perpendicular to (CD) from O cuts (xy) at E and (uv) at F.
Show that the two triangles OEC and OFD are congruent. **(4 points)**

12 FRACTIONS

Objectives

- Know the meaning of the terms : irreducible, reduced, simplify.
- Use the property $\frac{b}{b} = 1$ for any non-zero number b.
- Calculate the reduced form of a fraction using several methods.

CHAPTER PLAN

COURSE

EXERCISES AND PROBLEMS

TEST

1 FRACTIONS

• a and b are two integers where $b \neq 0$.

The writing $\frac{\boldsymbol{a}}{\boldsymbol{b}}$ is called a **fraction**.

The **numerator** $\boldsymbol{a}$ and the **denominator** $\boldsymbol{b}$ are the **terms** of the fraction $\frac{\boldsymbol{a}}{\boldsymbol{b}}$.

In particular : $\frac{a}{1} = a$; $\frac{0}{b} = 0$; $\frac{b}{b} = 1$.

EXAMPLES

$\frac{3}{7}$, $\frac{5}{8}$, $\frac{15}{17}$, $\frac{14}{21}$ and $\frac{121}{360}$ are fractions.

2 SIMPLIFYING FRACTIONS

Activity

1°) Complete : $\frac{18}{24} = \frac{18 : 3}{24 : 3} = \frac{\dots}{8}$.

$\frac{21}{35} = \frac{21 : 7}{35 : \dots} = \frac{\dots}{\dots}$.

2°) Given the fraction : $\frac{24}{36}$.

a) Is 4 a common divisor of 24 and 36?

b) Complete : $\frac{24}{36} = \frac{24 : 4}{36 : 4} = \frac{\dots}{\dots}$.

c) Give the simplest fraction equal to $\frac{6}{9}$.

Rule

To simplify a fraction $\frac{a}{b}$ is to replace it by an equal fraction, upon dividing its two terms by the same common divisor

Remark :

To simplify a fraction $\frac{a}{b}$, it is necessary to find the common divisors of the numerator and the denominator.

An integer is divisible by :

2 if it ends by 0 , 2 , 4 , 6 or 8 ;

5 if it ends by 0 or 5 ;

3 if the sum of its digits is divisible by 3 ;

9 if the sum of its digits is divisible by 9 ;

10 if it ends by 0.

EXAMPLES

$\frac{64}{20} = \frac{64 : 2}{20 : 2} = \frac{32}{10}$; $\frac{54}{63} = \frac{54 : 9}{63 : 9} = \frac{6}{7}$.

Application 1

1º) **a)** Simplify the fraction $\frac{175}{225}$ by dividing its terms by 5.

b) Can you simplify the obtained fraction ?

2º) Simplify each fraction :

$\frac{28}{42}$; $\frac{27}{63}$; $\frac{90}{126}$; $\frac{121}{66}$.

3º) **a)** Is 10 a common divisor of 210 and 360?

b) Complete :

$\frac{210}{360} = \frac{210 : 10}{360 : \dots} = \frac{\dots}{\dots}$

c) Simplify the fraction $\frac{21}{36}$.

3 REDUCIBLE FRACTION IRREDUCIBLE FRACTION

Activity

Given the fraction $\frac{32}{40}$.

Since 32 and 40 **are not relatively prime,** then $\frac{32}{40}$ is said to be a **reducible fraction**.
By simplifying the terms 32 and 40 ,
we obtain : $\frac{32}{40} = \frac{32:4}{40:4} = \frac{8}{10} = \frac{8:2}{10:2} = \frac{4}{5}$.

Since 4 and 5 are **relatively prime**, then $\frac{4}{5}$ is called an **irreductible fraction**.

Rule

Given the fraction $\frac{a}{b}$ ($b \neq 0$).

- **If a and b are not relatively prime, then $\frac{a}{b}$ is a reducible fraction.**
- **If a and b are relatively prime, then $\frac{a}{b}$ is irreductible.**
- **To reduce a fraction is to replace it by the irreductible fraction equal to it.**

EXAMPLES

- The fraction $\frac{345}{1275}$ is reducible since 5 is a common divisor of 345 and 1275.
- The fraction $\frac{14}{33}$ is irreducible since 14 and 33 are relatively prime.

  (1 is their only common divisor).

Application 2

1°) Indicate the irreducible fractions :
$\frac{5}{9}$; $\frac{18}{21}$; $\frac{32}{20}$; $\frac{7}{10}$; $\frac{4}{15}$; $\frac{7}{7}$; $\frac{41}{37}$.

2°) Reduce the fraction $\frac{315}{630}$.

4 PRACTICAL METHODS FOR REDUCING A FRACTION

Reduce the fraction $\dfrac{108}{144}$.

1) Method of successive divisions

$$\frac{108}{144} = \frac{108:2}{144:2} = \frac{54}{72} = \frac{54:2}{72:2} = \frac{27}{36}$$

$$= \frac{27:3}{36:3} = \frac{9}{12} = \frac{9:3}{12:3} = \frac{3}{4} .$$

2) Method of prime factorization

$108 = 2^2 \times 3^3$ and $144 = 2^4 \times 3^2$.

$$\frac{108}{144} = \frac{2^2 \times 3^3}{2^4 \times 3^2} = \frac{2 \times 2 \times 3 \times 3 \times 3}{\cancel{2} \times \cancel{2} \times 2 \times 2 \times \cancel{3} \times \cancel{3}} = \frac{3}{2 \times 2} = \frac{3}{4} .$$

3) Method using the G.C.F

$108 = 2^2 \times 3^3$ and $144 = 2^4 \times 3^2$.

G.C.D (108 and 144) $= 2^2 \times 3^2 = 36$.

$$\frac{108}{144} = \frac{108:36}{144:36} = \frac{3}{4},$$

$\dfrac{3}{4}$ is an irreducible fraction since 3 and 4 are relatively prime.

Application 3

1º) Simplify the following fraction : $\dfrac{51}{123}$; $\dfrac{105}{65}$; $\dfrac{100}{300}$; $\dfrac{1581}{2431}$.

2º) Reduce the fraction $\dfrac{216}{720}$.

3º) Find, in two different ways, the irreducible fraction equal to $\dfrac{1260}{1350}$.

5 FRACTIONS EQUAL TO AN IRREDUCIBLE FRACTION

$\frac{3}{7}$ is an irreducible fraction ; it is written :

$$\frac{3}{7} = \frac{3 \times 2}{7 \times 2} = \frac{3 \times 3}{7 \times 3} = \frac{3 \times 4}{7 \times 4} = \ldots = \frac{3 \times k}{7 \times k} \quad (k \neq 0).$$

Rule

Upon multiplying the two terms of an irreducible fraction by the same non zero whole number, a fraction equal to it is obtained.

$\frac{a}{b}$ is irreducible, therefore $\frac{a}{b} = \frac{a \times k}{b \times k}$ $(k \neq 0)$.

Application 4

Give four fractions equal to $\frac{3}{5}$.

$$\frac{a}{b} = \frac{a \times k}{b \times k} \quad (k \neq 0)$$

Exercises and problems

For testing the knowledge

1 Complete :

$\frac{49}{56} = \frac{\ldots}{8}$; $\frac{15}{25} = \frac{3}{\ldots}$; $\frac{6}{10} = \frac{54}{\ldots}$

$\frac{24}{\ldots} = \frac{4}{7}$; $\frac{5}{8} = \frac{\ldots}{40}$; $\frac{123}{\ldots} = \frac{3}{5}$.

2 Simplify the following fractions :

$\frac{4}{12}$; $\frac{45}{60}$; $\frac{140}{105}$; $\frac{30}{75}$; $\frac{300}{600}$;

$\frac{5 \times 11 \times 7}{7 \times 11}$; $\frac{5 \times 6^2 \times 11^2}{3^2 \times 7 \times 11}$;

$\frac{2^4 \times 3^2 \times 5}{12}$.

3 **1º)** Complete : $\frac{216}{720} = \frac{\dots}{360} = \frac{54}{\dots} = \frac{\dots}{90} = \frac{9}{\dots} = \frac{\dots}{\dots}$

2º) Give the irreducible fraction equal to $\frac{216}{720}$.

3º) Give three simplified fractions equal to $\frac{216}{720}$.

4 Give the irreducible fraction equal to each of the following fractions :

$\frac{30}{25}$; $\frac{90}{126}$; $\frac{500}{800}$; $\frac{42}{96}$.

5 Find the intruder in each case :

1º) $\frac{18}{20}$; $\frac{5}{13}$; $\frac{9}{27}$; $\frac{7}{42}$.   **2º)** $\frac{5}{7}$; $\frac{19}{21}$; $\frac{36}{45}$; $\frac{14}{17}$.

6 Calculate:

1º) $\frac{1}{2} - \frac{1}{3} + \frac{1}{6}$ **2º)** $\frac{4}{5} + \frac{2}{3} + \frac{7}{15}$ **3º)** $3 + \frac{3}{5} - \frac{3}{15}$

4º) $\frac{5}{2} + \frac{2}{3} - \frac{8}{6}$ **5º)** $\frac{7}{8} + \frac{5}{20}$ **6º)** $1 - \frac{1}{5}$

7º) $1 + \frac{1}{3} - \frac{1}{2}$ **8º)** $\frac{3}{12} - \frac{1}{6} - \frac{3}{36}$ **9º)** $5 - \frac{1}{2}$.

7 Calculate after simplifying :

1º) $\frac{2}{5} + \frac{8}{14} - \frac{6}{12}$; $\frac{15}{16} - \frac{12}{24} + \frac{5}{8}$; $\frac{2}{7} - \frac{18}{24} + \frac{4}{28} + 1$.

2º) $\frac{2}{3} \times \frac{4}{5}$; $5 \times \frac{3}{7}$; $\frac{4}{9} \times \frac{1}{2}$; $\frac{7}{6} \times 3$; $\frac{1}{4} \times \frac{5}{8}$; $\frac{3}{2} \times \frac{14}{9}$; $\frac{7}{6} \times \frac{4}{12}$.

3º) $\frac{2}{3} \div \frac{4}{5}$; $5 \div \frac{3}{7}$; $\frac{4}{9} \div \frac{1}{2}$; $\frac{7}{6} \div 3$; $\frac{1}{4} \div \frac{5}{8}$; $\frac{3}{2} \div \frac{14}{9}$; $\frac{7}{6} \div \frac{4}{12}$;

$\frac{5}{13} \div \frac{9}{13}$; $\frac{1}{3} \div 2$.

8 Write the fraction that corresponds to :

1º) Half of the third

2º) three quarters of the half

3º) the quarter of the quarter

4º) the fifth of the three halves.

9 Write the irreducible fractions having a denominator less than or equal to 8, and a numerator equal to 2.
Arrange these fractions in increasing order.

10 **1º)** Find the irreducible fraction equivalent to $\frac{52}{65}$.

2º) Complete :

$$\frac{52}{65} = \frac{\dots}{100} \ .$$

11 Find a and b if :

$$\frac{20}{36} = \frac{a}{9} \ ; \ \frac{a}{60} = \frac{3}{15} \ ; \ \frac{50}{b} = \frac{1}{2}$$

$$\frac{25}{125} = \frac{1}{b} \ ; \ \frac{12}{36} = \frac{4}{b} \ ; \ \frac{a}{270} = \frac{4}{9} \ .$$

12 **1º)** Using the method of successive divisions, find the irreducible fraction which is equal to $\frac{1026}{360}$.

2º) Find the G.C.F of 1026 and 360 ; deduce then the irreducible fraction equal to $\frac{1026}{360}$.

13 **1º)** Reduce the fraction $\frac{105}{195}$ by applying successive divisions to 105 and 195.

2º) Let d be the G.C.F of 105 and 195.

a) Calculate d .

b) Determine : $\frac{105 : d}{195 : d}$.

c) What can you say about the obtained fraction ?

14 Answer by true or false.

1º) If a and b are two natural numbers then $\frac{a}{b}$ is a fraction.

2º) $\frac{15}{20}$ is an irreducible fraction .

3º) A simplified writing of $\frac{12}{8}$ is $\frac{6}{9}$.

4º) $\frac{425}{325} = \frac{4}{3}$.

5º) $\frac{14}{20} = \frac{14-6}{20-6}$.

6º) To find the irreducible fraction equivalent to $\frac{240}{460}$, we divide the two terms 240 and 460 by their G.C.F.

For seeking

15 Reduce $\frac{564}{852}$ using the method of prime factorization.

16 **1°)** Are the fractions : $\frac{1}{2}$; $\frac{2}{3}$; $\frac{3}{4}$; $\frac{4}{5}$; $\frac{5}{6}$; $\frac{10}{11}$; $\frac{15}{16}$ and $\frac{23}{24}$ irreducible ?

2°) What can you deduce about the fraction $\frac{n}{n+1}$ where n is any non-zero natural number ?

17 Find the irreducible fraction equal to each of the following expressions :

1°) $\frac{60}{75} + 1$; **5°)** $\frac{5}{77} + \frac{4}{7}$

2°) $\frac{2}{3} + \frac{5}{6}$; **4°)** $\frac{5}{18} + \frac{8}{9}$

3°) $\frac{3}{4} - \frac{5}{8}$; **6°)** $\frac{1}{3} - 1 + \frac{8}{3}$.

18 Reduce the fraction $\frac{48}{80}$, then find its equivalent fraction having the sum of its terms 12.

19 Give the equivalent fraction of $\frac{68}{85}$, whose denominator is 100.

20 **1°)** Verify that the number 313131 is divisible by 31.

2°) Simplify the fraction $\frac{313131}{939393}$.

3°) Give the irreducible fraction equivalent to $\frac{313131}{939393}$.

21 Reduce the fraction $\frac{60}{252}$ then find its equivalent fraction whose terms have a sum of 130.

22 Find the G.C.F of a and b , then give the irreducible fraction equivalent to $\frac{a}{b}$ in each of the following cases :

1° $a = 540$ and $b = 60$.
2° $a = 612$ and $b = 828$.
3° $a = 2205$ and $b = 3675$.
4° $a = 3600$ and $b = 5920$.

23 Calculate :

1°) $\frac{4}{5} \times \frac{5}{2} + \frac{3}{2} \times 4$.

2°) $4 + 5 \times \frac{4}{3} - 2 \times \frac{5}{3}$.

3°) $\frac{2}{3} + \frac{5}{3} \times 8$.

4°) $\left(\frac{1}{2} - \frac{1}{3}\right) \div \left(1 + \frac{2}{3}\right)$.

5°) $1 - \frac{1}{3} \div \left(1 + \frac{2}{3}\right)$.

6°) $\left(\frac{3}{4} - \frac{49}{6}\right) \div \frac{2}{3}$.

24 **1°)** Calculate the sum and the product of $\frac{3}{7}$ and $\frac{7}{4}$.

2°) Calculate the sum of the reciprocal of $\frac{3}{7}$ and $\frac{7}{4}$.

25 Calculate for $a = \frac{1}{2}$ and $b = \frac{3}{7}$:

1°) $3a - 2b$.
2°) $-2a + b + 1$.
3°) $a + 3b$.

TEST

1 Complete : $\frac{42}{56} = \frac{21}{...} = \frac{...}{4}$ **(1 point)**

2 Is the fraction $\frac{242361}{111111}$ irreducible? Justify . **(1 point)**

3 Determine x knowing that :

1º) $\frac{30}{70} = \frac{x}{7}$; **2º)** $\frac{225}{x} = \frac{5}{3}$; **3º)** $\frac{125}{1200} = \frac{5}{x}$. **(3 points)**

4 **a)** Write as a product of prime factors the numbers 1458 and 2187.

b) Find the G.C.F of 1458 and 2187.

c) Give the irreducible fraction equivalent to $\frac{1458}{2187}$. **(3 points)**

5 Indicate among the following fractions, those that are reducible :

$\frac{14}{28}$; $\frac{17}{19}$; $\frac{15}{24}$; $\frac{19}{30}$; $\frac{20}{21}$; $\frac{555}{999}$. **(3 points)**

6 Find the irreducible fraction equivalent to $1 + \frac{12}{384}$. **(1 point)**

7 Reduce the fraction $\frac{540}{288}$ then find its equivalent fraction having the difference of its terms 14 . **(2 points)**

8 Find the correct answer and write its corresponding letter.
You will find out a word useful in certain calculations.

1. $1 \div \frac{1}{3} =$	N $\frac{1}{3}$	G 1	S 3	M $\frac{2}{3}$
2. $4 \times \left[15 - 2 \times (7 + 3) + 5\right] =$	O 180	I 0	L 36	E 4
3. $\frac{65}{3} + \frac{13}{3} =$	P $\frac{78}{6}$	S $\frac{52}{3}$	I $\frac{78}{9}$	G 26
4. $\frac{2 + 15}{6} - \frac{5}{3} =$	I 2	N $\frac{7}{6}$	M $\frac{19}{3}$	E $\frac{17}{3}$
5. $32 - \frac{17}{4} \times 2 =$	E $\frac{47}{2}$	P 6	I $\frac{15}{2}$	S $\frac{111}{2}$

(6 points)

13 DECIMAL FRACTIONS

Objectives

- Write a decimal fraction in the form of a decimal number.
- Write a decimal number as the sum of several fractions whose denominators are 10, 100, 1000, ...
- Define and recognize a non-decimal fraction.
- Know that a non-decimal fraction can be written as a decimal having an infinite number of repetitive digits after the point.
- Calculate an approximate value of a non-decimal fraction.

CHAPTER PLAN

COURSE

1 - Quotient of two numbers

2 - Property

3 - Rational number

4 - Decimal fraction

EXERCISES AND PROBLEMS

TEST

COURSE

1 QUOTIENT OF TWO NUMBERS

- a and b are two numbers where $b \neq 0$. The **quotient** of a and b is denoted by :

  $\boldsymbol{a : b}$ or $\boldsymbol{a \div b}$ (a divided by b) or $\dfrac{\boldsymbol{a}}{\boldsymbol{b}}$ (a over b) ; a is the numerator and

  b is the denominator. $\dfrac{\boldsymbol{a}}{\boldsymbol{b}}$ **is a fractional writing.**

- If the division of a by b **«ends»**, then the quotient is a **decimal number.**

EXAMPLE

$$\begin{array}{r|l} & \;6.4 \\ \hline 5 & \;32 \\ & -30 \\ \cline{2-2} & \;20 \\ & -20 \\ \cline{2-2} & \;\;0 \end{array} \qquad\qquad \begin{array}{r|l} & \;3.666\ldots \\ \hline 3 & \;11 \\ & -9 \\ \cline{2-2} & \;20 \\ & -18 \\ \cline{2-2} & \;\;2 \end{array}$$

The quotient $\dfrac{32}{5}$ admits the decimal writing : 6.4.

$\dfrac{11}{3}$ is not a decimal. 3.6 ; 3.66 ; 3.666 are **approximations** of $\dfrac{11}{3}$.

- If a and b are **whole** numbers, the quotient $\dfrac{\boldsymbol{a}}{\boldsymbol{b}}$ is a **fraction**.

EXAMPLE

$\dfrac{3}{4}$ is a fraction.

$\dfrac{0.3}{4}$ is not a fraction since 0.3 is not a whole number.

Application 1

1º) Among the following, which are fractions ? Justify .

$\frac{2.3}{7}$; 4; $\frac{3}{7}$; $\frac{1}{1.8}$.

State why the others are not fractions.

2º) Calculate, for each of the following fractions, the decimal writing or the approximated writing rounded to the nearest hundredth .

$\frac{2}{5}$; $\frac{3}{7}$; $\frac{7}{-2}$; $\frac{-8}{10}$; $\frac{41}{14}$.

2 PROPERTY

Activity

a) Divide 12 by 2.5 then complete :

$\frac{12}{2.5} = \ldots$

b) Calculate $\frac{12 \times \mathbf{2}}{2.5 \times \mathbf{2}}$ then $\frac{12 : \mathbf{2}}{2.5 : \mathbf{2}}$.

What do you notice ?

Rule

The value of a quotient $\frac{a}{b}$ **does not change** if its numerator and denominator are **multiplied** or **divided** by the same non-zero number.

EXAMPLES

- $\frac{32}{5} = 6.4$ and $\frac{32 \times \mathbf{2}}{5 \times \mathbf{2}} = \frac{64}{10} = 6.4$
- $\frac{30}{40} = 0.75$ and $\frac{30 : \mathbf{2}}{40 : \mathbf{2}} = \frac{15}{20} = 0.75$

Remark :
The property above enables each quotient to be written as a fraction.

EXAMPLES

- $\dfrac{3.2}{4} = \dfrac{3.2 \times \mathbf{10}}{4 \times \mathbf{10}} = \dfrac{32}{40}$.
- $\dfrac{0.01}{2.3} = \dfrac{0.01 \times \mathbf{100}}{2.3 \times \mathbf{100}} = \dfrac{1}{230}$.
- $\dfrac{-0.3}{4} = \dfrac{-0.3 \times \mathbf{10}}{4 \times \mathbf{10}} = \dfrac{-3}{40} = -\dfrac{3}{40}$.

Application 2

1º) Find the irreducible fraction equivalent to each of the following fractions :
$\dfrac{180}{40}$; $\dfrac{91}{26}$; $\dfrac{121}{77}$; $\dfrac{105}{140}$.

2º) Write each of the following quotients in the form of a simplified fraction :
$\dfrac{10.5}{14}$; $\dfrac{-0.12}{5.6}$; $\dfrac{80}{3.6}$; $\dfrac{15.2}{-54}$.

3 THE RATIONAL NUMBERS

Definition

A rational is a number that can be written in the form of $\dfrac{a}{b}$ where a is an integer and b is a non-zero integer.

- The decimal –3.2 is a rational since it can be written in the form: $\dfrac{-32}{10}$ $\left(\dfrac{a}{b}\right.$ where a is an integer and b is a non-zero integer$\left.\right)$.

Every decimal is a rational.

• The natural number 7 is a rational since it can be written in the form of: $\frac{7}{1}$.

Every natural number is a rational.

• The number 3.666.... where 6 is a repetitive, is called an infinite periodic number; it is a rational that is written $\frac{11}{3}$.

The number 12.3141414... where 14 is repetitive is called an infinite periodic number; it is a rational that is written $\frac{12\,191}{990}$.

Any periodic number is a rational.

• The infinite non-periodic number 3.1415927... is an approximation of π ; it is not a rational since it cannot be written in the form $\frac{a}{b}$ where a and b are non-zero whole numbers.

Any infinite non-periodic number is not a rational.

Application 3

Name among the following the rational numbers.

$\frac{7}{3}$; -8.3 ; 7.636363… ; 2.15 ; 3.7654317

0 ; $\frac{-4}{9}$; -8 ; 10 ; $\frac{60}{10}$.

4 DECIMAL FRACTION

Definition

Any fraction $\frac{a}{b}$ where the **division** of a by b «**ends**» is called a **decimal fraction**. Such a fraction can be written in the form of a fraction having its **denominator** a **power of 10**.

EXAMPLES

- $\dfrac{7}{5}$ is a decimal fraction since 7 : 5 = 1.4 (decimal).

  $\dfrac{7}{5} = \dfrac{7 \times 2}{5 \times 2} = \dfrac{14}{10}$.

- $\dfrac{11}{3}$ is not a decimal fraction since the division of 11 by 3 « does not end»;

  $\dfrac{11}{3} = 3.66666\ldots$

  It cannot be written in the form of a fraction whose denominator is a power of 10.

- $\dfrac{75}{40}$ is a decimal fraction, 75 : 40 = 1.875.

  $\dfrac{75}{40} = \dfrac{75 : 5}{40 : 5} = \dfrac{15}{8} = \dfrac{15 \times 125}{8 \times 125} = \dfrac{1\,875}{1\,000} = \dfrac{1\,875}{10^3}$.

Practical methods to recognize whether a fraction is a decimal or not.

1st method

Simplify the fraction so that it becomes irreducible.
If the obtained fraction can be written in the form of a fraction whose denominator is a power of 10, then the given fraction is a decimal.

EXAMPLES

- The fraction $\dfrac{42}{84}$ is written :

  $\dfrac{42}{84} = \dfrac{42 : 42}{84 : 42} = \dfrac{1}{2} = \dfrac{1 \times 5}{2 \times 5} = \dfrac{5}{10}$;

  it is therefore a decimal fraction. 0.5 is its decimal writing.

- The fraction $\dfrac{24}{40}$ is written :

  $\dfrac{24}{40} = \dfrac{24 : 8}{40 : 8} = \dfrac{3}{5} = \dfrac{3 \times 2}{5 \times 2} = \dfrac{6}{10}$;

  it is therefore a decimal fraction. 0.6 is its decimal writing.

• The fraction $\frac{15}{200}$ is written : $\frac{15}{200} = \frac{15:3}{200:5} = \frac{3}{40} = \frac{3 \times 25}{40 \times 25} = \frac{75}{1\,000}$;

it is therefore a decimal fraction. 0.075 is its decimal writing.

• The fraction $\frac{57}{42}$ is written : $\frac{57}{42} = \frac{57:3}{42:3} = \frac{19}{14}$;

hence, it is not a decimal.

2nd method

Reduce the given fraction. Write the denominator of the obtained fraction as a product of prime factors.
If only 2 or 5 are obtained as prime factors, then the fraction is decimal.

EXAMPLES

• $\frac{42}{84} = \frac{42:42}{84:42} = \frac{1}{\mathbf{2}}$; $\mathbf{2 = 2^1}$, then $\frac{42}{84}$ is a decimal fraction.

It is written : 0.5.

• $\frac{24}{200} = \frac{24:8}{200:8} = \frac{3}{\mathbf{25}}$; $\mathbf{25 = 5^2}$, then $\frac{24}{200}$ is a decimal fraction.

It is written : 0.12.

• $\frac{15}{200} = \frac{15:5}{200:5} = \frac{3}{\mathbf{40}}$; $\mathbf{40 = 2^3 \times 5}$, then $\frac{15}{200}$ is a decimal fraction.

It is written : 0.075.

• $\frac{57}{210} = \frac{57:3}{210:3} = \frac{19}{\mathbf{70}}$; $\mathbf{70 = 2 \times 5 \times 7}$, then $\frac{57}{210}$ is not a decimal fraction.

Application 4

1º) Show that the following fractions are decimal fractions (write each one in the form of a fraction whose denominator is a power of 10)

$\frac{2}{100}$; $\frac{1}{500}$; $\frac{51}{60}$; $\frac{63}{75}$; $\frac{81}{36}$.

2º) Give the decimal writing of each of the fractions of 1º)

Non decimal fraction

Every non-decimal fraction is written in the form of a **number with a decimal point**, in which the decimal part is **repeated** or **periodic.**

EXAMPLES

- $\dfrac{37}{3}$ is not a decimal fraction ; in fact, the division of 37 by 3

  «does not end». $37 : 3 = 12.\underline{3}\ \underline{3}\ \underline{3}\ \underline{3}\ ...$

  12.3333... is a number where the decimal part is repeated. It is a rational but not a decimal.

- $\dfrac{49}{6}$ is not a decimal fraction ;

  in fact $49 : 6 = 8.1\ \underline{6}\ \underline{6}\ \underline{6}\ \underline{6}\ ...$

  8.16666.... is a number where the decimal part is repeated; it is a rational but not a decimal.

- Similarly for the fraction $\dfrac{71}{99}$ which is equal to $0.\ \underline{71}\ \underline{71}\ \underline{71}\ ...$

Application 5

1°) Simplify each of the following fractions below. Decompose into prime factors the denominator of the obtained fraction and deduce if it is a decimal or not.

$\dfrac{46}{36}$; $\dfrac{5}{75}$; $\dfrac{35}{300}$; $\dfrac{21}{420}$; $\dfrac{6}{30}$; $\dfrac{35}{20}$; $\dfrac{26}{42}$.

2°) Give the decimal writing or the approximate value of the fractions of 1°), to the nearest hundredth.

EXERCISES AND PROBLEMS

For testing the knowledge

1 Complete:

$\dfrac{7}{2} = \dfrac{\dots}{4}$; $\dfrac{17}{21} = \dfrac{68}{\dots}$; $\dfrac{12}{13} = \dfrac{36}{\dots}$; $\dfrac{75}{50} = \dfrac{\dots}{2}$; $\dfrac{8}{12} = \dfrac{\dots}{3}$

$\dfrac{5}{7} = \dfrac{15}{\dots} = \dfrac{\dots}{28} = \dfrac{30}{\dots} = \dfrac{\dots}{77}$.

2 Give a fractional writing for each of the following quotients :

$13 \div 11$; $4.5 \div 2$; $1 \div 32$.

3 Write each of the following quotients in the form of a fraction.

$\frac{0.1}{0.3}$; $\frac{1.7}{1.3}$; $\frac{2.2}{0.11}$; $\frac{0.45}{9}$; $\frac{1.4}{0.07}$.

4 Write the irreducible fraction.

$\frac{20}{12}$; $\frac{56}{72}$; $\frac{140}{105}$; $\frac{108}{84}$; $\frac{28}{98}$; $\frac{75}{100}$; $\frac{12}{100}$; $\frac{125}{100}$.

5 Give the decimal writing of each of the following quotients :

$\frac{7}{20}$; $\frac{23}{5}$; $\frac{5.3}{53}$; $\frac{3.5}{5}$.

6 The following divisions «have an end».

Perform the operation and give the decimal writing of each quotient.

12.45 by 15 ; 3.23 by 1.9 ; 42.9 by 8.25 .

7 Give, when possible, the decimal writing of each of the following fractions.

$\frac{13}{15}$; $\frac{26}{39}$; $\frac{2}{18}$; $\frac{4}{32}$; $\frac{27}{2}$.

8 **1º)** Find a fraction which has 0.25 as a decimal writing.

2º) Give the decimal writing of $\frac{2}{5}$.

9 Write in the form of an irreducible fraction.

0.0035 ; 0.7 ; 5.4 ; 0.08 ; 13 ; 11.32 .

10 Observe and complete .

$$\mathbf{12.345 = 12 + \frac{3}{10} + \frac{4}{100} + \frac{5}{1\,000} = 12 + 0.3 + 0.04 + 0.005}$$

1º) 7.46 = ……… **3º)** 0.001 = ………

2º) 1.036 = ……… **4º)** 0.4 = …..……

11 Write in the form of an irreducible fraction each of the following numbers.

$4 + \frac{2}{10}$; $3 + \frac{1}{10}$; $0.1 + \frac{31}{100}$; $\frac{23}{1\,000} + \frac{2}{100}$; $25 + \frac{9}{10} + \frac{9}{1\,000}$.

12 Answer by true or false.

1º) We do not change the quotient $\frac{a}{b}$ if we add the same number to the numerator and to the denominator .

2º) $\frac{8}{0.4} = 0.2$.

3º) $\frac{2}{7}$ is a decimal fraction.

4º) $\frac{19}{2}$ is not a decimal fraction.

5º) The quotient of two whole numbers is always a whole number.

6º) Every decimal number can be written in the form of a fraction.

7º) $\frac{3}{7} = \frac{21}{49}$.

8º) $12.4502 = 12 + \frac{4}{10} + \frac{50}{100} + \frac{2}{1\,000}$.

9º) $\frac{2.4}{4}$ is a decimal fraction.

10º) $\frac{707}{99} = 7.\,14\,14\,14...$, then $\frac{707}{99}$ is a decimal.

For seeking

13 Write in the form of a fraction.

$12 + 3 \div 4$; $(12 + 3) \div 4$; $12 \div (3 + 4)$; $(23 + 7) \div (2 + 5)$.

14 Here are the answers of Zahi on a quiz on equal fractions. Each answer scores 2 points if it is correct and 0 if it is wrong :

$\frac{2}{7} = \frac{20}{70}$; $\frac{4}{12} = \frac{0}{3}$; $\frac{5}{30} = 6$; $\frac{2}{3} = \frac{4}{9}$; $\frac{15}{10} = \frac{3}{2}$.

What is Zahi's grade ?

15 Simplify the following fractional writings :

$\frac{4.5}{3.5}$; $\frac{2.1}{1.2}$; $\frac{24}{5.4}$; $\frac{0.42}{4.3}$.

16 **1º)** Perform the division of 25 by 7.

2º) Give two framings of $\frac{25}{7}$, to the nearest 0.01 and to the nearest 0.0001 .

17 Use the calculator to calculate $\frac{23.5}{\pi}$ (where $\pi = 3.14$).

Read the displayed result and complete.

1º) $7 < \frac{23.5}{\pi} < \ldots$ to the nearest whole number,

2º) $\ldots < \frac{23.5}{\pi} < \ldots$ to the nearest hundredth,

3º) $\ldots < \frac{23.5}{\pi} < \ldots$ to the nearest tenth,

4º) $\ldots < \frac{23.5}{\pi} < \ldots$ to the nearest thousandth.

18 Consider the fractions : $\frac{2 \times 3}{2^2 \times 3 \times 5}$; $\frac{7 \times 5^2 \times 3}{2 \times 5 \times 3}$; $\frac{2 \times 11 \times 5}{2^2 \times 11^2}$.

1º) Simplify them.

2º) Find which one of the above fractions is not decimal. Give its approximate value to the nearest tenth.

3º) Give a decimal writing to each of the following decimal fractions.

TEST

1 Write in the form of a sum of decimal fractions.

2.5 ; 1.7603 . **(4 points)**

2 Write each of the following decimals in the form of an irreducible fraction.

5.64 ; 0.03 ; 12.4 ; 76.002 . **(6 points)**

3 Consider the quotients :

$$\frac{80}{400} \; ; \; \frac{1.3}{4.5} \; ; \; \frac{0.5}{2.4} \; ; \; \frac{3}{21} \; ; \; \frac{15}{64} \; ; \; \frac{0.2}{12.5} \; ; \; \frac{14}{35} \; ; \; \frac{2.4}{3} .$$

1º) Write them in the form of fractions and simplify them.

2º) List among the simplified fractions those which are decimals and give their decimal writings.

3º) Give the approximate value to the nearest hundredth for each non decimal fraction. **(4 - 3 - 3 points)**

14 CONGRUENT TRIANGLES (2)

Objective

Knowing that if two triangles have an angle and its adjacent sides respectively equal then these triangles are congruent.

CHAPTER PLAN

COURSE

1 - Second case of the congruency of triangles
2 - Commentary exercise

EXERCISES AND PROBLEMS

TEST

1 SECOND CASE OF THE CONGRUENCY OF TRIANGLES

Activity

a) Draw an angle $\widehat{xEy}$ of measure 65º ; place the point F on $[Ex)$ and the point G on $[Ey)$ such that $EF = 4$ cm and $EG = 7$ cm ; join F and G ; measure $[FG]$.

You have drawn therefore a triangle EFG knowing an angle and its adjacent sides.

Which side is opposite to angle $\widehat{FEG}$?

Which angle is opposite to side $[EF]$? $[EG]$?

b) Similarly draw a triangle MNP such that : $MN = 4$ cm, $MP = 7$ cm and $\widehat{NMP} = 65º$.

c) Copy each of the two triangles drawn above.

d) Verify that these two copies are congruent.

e) Determine in these two triangles :

1º) the equal angles.

2º) the congruent sides.

Rule

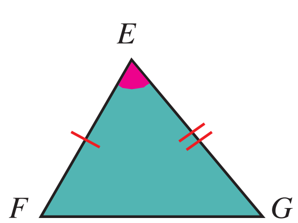
If two sides and the included angle of one triangle are respectively equal to two sides and the included angle of a second triangle, then the two triangles are congruent

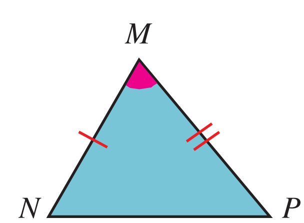
EXAMPLE

Consider the two triangles EFG and MNP such that :

$EF = MN$, $EG = MP$ and $\widehat{FEG} = \widehat{NMP}$;

Thus, these triangles are congruent (this is verified in the activity)





Application

$[AB]$ and $[CD]$ are two segments intersecting at their common midpoint O.

Show that $AC = BD$.

2 COMMENTARY EXERCISE

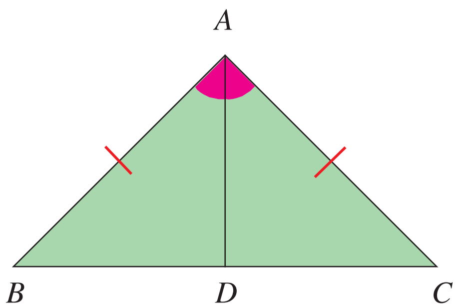
Let ABC be an isosceles triangle of vertex A.

The bisector of angle $\widehat{BAC}$ cuts the base $[BC]$ at D.

Show that :

1°) $BD = CD$,

2°) $\widehat{ADB} = \widehat{ADC} = 90°$.



<u>Given</u>

$AB = AC$

$\widehat{BAD} = \widehat{CAD}$

<u>Required to prove</u>

• $DB = DC$

• $\widehat{BDA} = \widehat{CDA} = 90°$.

<u>PROOF</u>

1°) Consider the two triangles ABD and ACD ; they have :

$AB = AC$ *(given)*,

$\widehat{BAD} = \widehat{CAD}$ *(given)*,

$[AD]$ common side.

These two triangles are congruent since the two sides and the included angle of one are equal to the sides and the included angle of the second.

All their corresponding parts are also equal, in particular :

$DB = DC$ ($[DB]$ and $[DC]$ are opposite to the equal angles $\widehat{BAD}$ and $\widehat{CAD}$) that is :

$[AD]$ is the median relative to the base [BC].

2°) $\widehat{ADB} = \widehat{ADC}$ (opposite to the congruent sides [AB] and [AC]).

But : $\widehat{ADB} + \widehat{ADC} = 180°$, then :

$\widehat{ADB} = \widehat{ADC} = \dfrac{180°}{2} = 90°$,

that is (AD) is perpendicular to (BC) $\big((AD) \perp (BC)\big)$.

[AD] is the height relative to the base [BC].

From the preceding exercise we can state the following properties :

In an **isosceles triangle** :

the **bisector** of the vertex angle is at the same time the **median** and the **height** relative to the base ; it is therefore, the **perpendicular bisector** of the base.

Exercises and problems

For testing the knowledge

1 Draw a triangle CAR in each of the following cases :

a) $AC = 30$ mm ; $AR = 42$ mm and $\widehat{CAR} = 50°$.

b) $\widehat{ARC} = 70°$, $RA = 4$ cm and $RC = 45$ mm.

c) $CA = 4$ cm , $CR = 3$ cm and CAR right at C.

2 Two segments [AB] and [CD] intersect at O such that :
$OB = OC$ and $OD = OA$.

Prove that the two triangles AOC and BOD are congruent;
list the equal angles.

3 In triangle ABC, we produce the median $[AM]$ to a length MA' such that $MA' = MA$.

Prove that the two triangles AMB and $A'MC$ are congruent.

4 SEC is any triangle. O is the symmetric of E with respect to S ; L is the symmetric of C with respect to S ; D is the midpoint of $[EC]$ and D' the midpoint of $[OL]$.

a) Compare OL and EC.

b) Compare SD and SD'.

5 ABC and $A'B'C'$ are two congruent triangles; the corresponding sides are $[AB]$ and $[A'B']$, $[AC]$ and $[A'C']$, $[BC]$ and $[B'C']$.
$[AM]$ and $[A'M']$ are the medians relative to $[BC]$ and $[B'C']$ respectively.

Prove that $AM = A'M'$.

6 ABC is an isosceles triangle of base $[BC]$. The bisector of angle $\widehat{ABC}$ cuts $[AC]$ at B' and that of $\widehat{ACB}$ cuts $[AB]$ at C'.

Prove that $BB' = CC'$.

7 Answer by true or false.
Two triangles ABC and $A'B'C'$ are congruent if :

1º) $AB = A'B'$, $AC = A'C'$ and $\widehat{ABC} = \widehat{A'B'C'}$.

2º) $AC = A'C'$, $BC = B'C'$ and $\widehat{BAC} = \widehat{B'A'C'}$.

3º) $\widehat{BAC} = \widehat{B'A'C'}$, $\widehat{ABC} = \widehat{A'B'C'}$ and $\widehat{ACB} = \widehat{A'C'B'}$.

4º) $AB = A'B'$, $AC = A'C'$ and $\widehat{BAC} = \widehat{B'A'C'}$.

5º) $BC = B'C'$, $\widehat{ABC} = \widehat{A'B'C'}$ and $\widehat{ACB} = \widehat{A'C'B'}$.

6º) $AC = A'C'$, $BC = B'C'$ and $\widehat{ACB} = \widehat{A'C'B'}$.

For seeking

8 $[Ou)$ is the bisector of an angle $\widehat{xOy}$. A is a point of $[Ox)$ and B is a point of $[Oy)$ such that $OA = OB$. M is any point of $[Ou)$.

a) Prove that the two triangles OAM and OBM are congruent. Deduce that $[MO)$ is the bisector of $\widehat{AMB}$.

b) The perpendicular drawn from M to $[Ou)$ cuts $[Ox)$ at C and $[Oy)$ at D.
Prove that $OC = OD$ and that $AC = BD$.

9 ABC is an isosceles triangle of vertex A ; $[BM]$ and $[CN]$ are the medians relative to the sides $[AC]$ and $[AB]$ respectively.

a) Prove that the two triangles AMB and ANC are congruent.
Deduce that $CN = BM$ and that $\widehat{ACN} = \widehat{ABM}$.

b) $[BM]$ and $[CN]$ intersect at I.
Show that triangle IBC is isosceles of vertex I.
Deduce that IMN is an isosceles triangle.

10 ABC is an isosceles triangle of vertex A. I is the midpoint of $[BC]$. P is a point of $[BI]$ and Q a point of $[CI]$ such that $BP = CQ$.

a) Prove that I is the midpoint of $[PQ]$.

b) The perpendiculars at P and Q to $[BC]$ cut $[AB]$ at M and $[BC]$ at N respectively.
Prove that $BM = CN$.

c) Prove that triangle MIN is isosceles.

11 ABC is an isosceles triangle of vertex A. The bisector of angle $\widehat{BAC}$ cuts $[BC]$ at M.

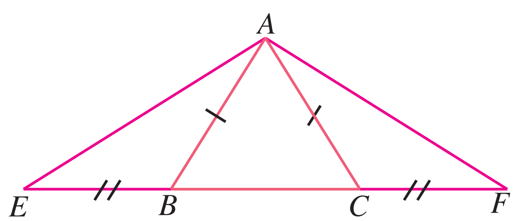
a) N is a point of $[AM]$; Prove that the two triangles ANB and ANC are congruent.
Deduce that triangle NBC is isosceles.

b) Let E be a point of $[MA)$ such that A belongs to $[ME]$.
Prove that the two triangles EAB and EAC are congruent.

12 MNP is any given triangle. On the bisector $[Mx)$ of angle $\widehat{NMP}$, we consider the points E and F such that $ME = MN$ and $MF = MP$.

Prove that $NF = PE$.

13 In the adjacent figure, ABC is an isosceles triangle and $BE = CF$.



1º) Prove that the two triangles ABE and ACF are congruent .

2º) Deduce that triangle AEF is isosceles.

3º) The bisector of angle $\widehat{BAC}$ cuts (BC) at I .
Prove that $[AI)$ is the bisector of angle $\widehat{EAF}$.

TEST

1 ABC is an isosceles triangle of vertex A. E and F are two points of $[BC]$ such that $BE = CF$.

Prove that triangle AEF is isosceles. **(3 points)**

2 From the vertex O of angle $\widehat{xOy}$ and to the exterior of this angle, we draw $[Ox')$ perpendicular to $[Ox)$ and $[Oy')$ perpendicular to $[Oy)$. A is a point of $[Ox)$ and B is a point of $[Ox')$ such that $OA = OB$. C is a point of $[Oy)$ and D is a point of $[Oy')$ such that $OC = OD$.

a) Compare angles $\widehat{AOD}$ and $\widehat{BOC}$. **(3 points)**

b) Prove that $AD = BC$. **(3 points)**

3 ABC is an isosceles triangle of vertex A. The perpendiculars drawn from B and C to (AB) and (AC) respectively intersect at M.

a) Prove that $\widehat{MBC} = \widehat{MCB}$. **(2 points)**

b) (BM) cuts (AC) at E and (CM) cuts (AB) at F. Prove that the two triangles BCF and BCE are congruent. **(3 points)**

4 Given an angle $\widehat{xOy}$. A and C are two points of $[Ox)$, B and D are two points of $[Oy)$ such that $OA = OB$ and $OC = OD$. $[AD]$ and $[BC]$ intersect at I.

a) Prove that the two triangles OAD and OBC are congruent. Deduce that $AD = BC$ and $\widehat{CAI} = \widehat{DBI}$. **(3 points)**

b) Prove that the two triangles IAC and IBD are congruent. **(3 points)**

15 ALGEBRAIC EXPRESSIONS

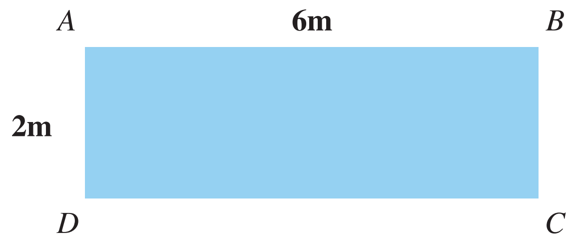
Objectives

- Knowing the definition of the term algebraic or monomial, coefficient, variable, algebraic expression.
- Finding the like terms in an algebraic expression.
- Reducing the like terms in an algebraic expression.
- Performing calculation on the algebraic expression (addition, subtraction, multiplication...)

CHAPTER PLAN

1 DEFINITIONS

Activity



1°) A rectangular field *ABCD* has for dimensions 6 m and 2 m.
Calculate the area of this field.

2°) Designate by L and l the dimensions of *ABCD*. Express the area $\mathcal{A}$ in terms of L and l.

3°) What does the area $\mathcal{A}$ become by taking as length $2 \times a$ and as width $3 \times b$?

Algebraic expression

In the goal of simplifying and to generalizing questions that can be asked on numbers, they are frequently represented by letters.

An **algebraic expression** is a collection of letters called variables and numbers organized in some manner by using the operations (+ ; – ; × ; ÷).

$3 \times x^2$; $6 \times a^2 \times b$; $3 \times x^2 \times y + 5 \times x^3$; $6 \times a^2 \times c - 8 \times b^3 \times c^2$ and $4 \times x^2 \div 3 \times y$ are **algebraic expressions**.

In $3 \times x^2 \times y + 5 \times x^3$, $3 \times x^2 \times y$ and $5 \times x^3$ are **the terms** of this expression.

Monomial

Each term in an algebraic expression is called a **monomial** :

In the algebraic expression $8 \times x^2 \times y - 3 \times x \times y^5$, $8 \times x^2 \times y$ and $-3 \times x \times y^5$ are monomials.

• In the monomial $2 \times x^3$, 2 is called the **coefficient** and x the **variable**.

• In the monomial $-2.5 \times x^2 \times y$, -2.5 is the **coefficient**, x and y are the **variables**.

Simplified writing

• The **literal** writing of $a \times b$ is ab. a and b are the factors of the product ab.

• a being an integer and n a natural number, we write :

$$\underbrace{a + a + ... + a}_{n \text{ terms}} = na$$

Also : $a + a = 2a$; $a + a + a = 3a$; etc ...

Remark : $5 + 5 + 5$ is written **3 × 5 and not 35** .

Examples

• The monomial $2 \times x^3$ is written $2x^3$

• The monomial $\frac{-5}{2} \times x^3 \times y$ is written $\frac{-5}{2} x^3y$

• The algebraic expression $6 \times a^2 \times c - 8 \times b^3 \times c^2$ is written $6a^2c - 8b^3c^2$.

• The monomial $1 \times a$ is written a .

• The monomial $-1 \times a$ is written $-a$.

Application 1

Complete the following table :

Monomial	Variable	Coefficient	Exponent of the variable
$-5x^3$			
	y	-1.5	2
x^8			
	a	2	5
$3y^2$			
	t	-3.5	1

Like terms

We call like terms the terms that differ only by their cœfficients.

EXAMPLES

- $7x^6$ and $-3x^6$ are two like terms.
- $-3x^2y^3$, $-5x^2y^3$ and $10x^2y^3$ are not like terms.
- $4a^2b$ and $-3ab^2$ are not like terms.
- $8x^3$ and $-3x^2$ are not like terms.

Application 2

Collect the like terms :

$3x^5$; $-2a^2b^3$; $6x^5y^2$; $-4x^5$; $-1.5t^3$;

$\frac{3}{4}a^2b^3$; $-\frac{1}{5}x^5y^2$; $\frac{4}{5}t^3$; $2xy^2z^3$; $-0.5a^2b^3$.

Numerical value

The numerical value of an algebraic expression is the result obtained by replacing the letters by given numbers and performing the given operations.

EXAMPLE

The numerical value of the algebraic expression $3x^2y + 5x^3$ for $x = 2$ and $y = 3$ is :

$3 \times 2^2 \times 3 + 5 \times 2^3 = 36 + 40 = 76.$

Application 3

Calculate the numerical value of the algebraic expression $6a^2c - 8b^3c^2$ for $a = -1$, $b = 2$ and $c = 1.5$.

2 MULTIPLICATION OF MONOMIALS

Activity

1º) Complete the following table :

x	2	-1	0.3
x^2			
$3x^2$			
x^3			
$2x^3$			
$3x^2 \times 2x^3$			
x^5			
$6x^5$			

2º) Compare the pink lines and complete :

$$\mathbf{3x^2 \times 2x^3 = \ldots\ldots\ldots\ldots\ldots\ldots}$$

Rule

The product of two or more monomials is obtained by multiplying their cœfficients and adding the exponents of the same variable.

EXAMPLES

- $2x^3 \times (-3x^5) = -6x^8$
- $3a^2b \times (-5ab^3) = -15a^3b^4$
- $xy \times 2ay^3 \times (-3a^2x^4y^4) = -6a^3x^5y^8.$

Application 4

Perform :

1°) $-4x^3 \times 2x^4 \times x$ **2°)** $3ab^2 \times (-2a^2b) \times 5c$.

3 CALCULATION OF ALGEBRAIC EXPRESSIONS

Activity

1°) Complete the following table :

x	2	–3	1.5
$2x^2$			
$5x^2$			
$2x^2 + 5x^2$			
$7x^2$			
$2x^2 - 5x^2$			
$-3x^2$			

2°) Compare the pink lines and complete :

$$2x^2 + 5x^2 = \text{........................}$$

3°) Compare the yellow lines and complete :

$$2x^2 - 5x^2 = \text{........................}$$

Reducing like terms

To reduce like terms in an algebraic expression is to replace them by a unique term, simply by adding or subtracting their cœfficients.

EXAMPLES

- $3x^2 - 4x^2 + 8x^2 = (3 - 4 + 8)\ x^2 = 7x^2$.
- $a^2b^4 + 3a^2b^4 - 0.5a^2b^4 = (1 + 3 - 0.5)\ a^2b^4 = 3.5\ a^2b^4$.

Application 5

Reduce the like terms in each of the following algebraic expressions (a, b, m, x and y are variables).

1º) $3x^2 + 4y^2 - 5x^2 + y^2 + x^2 - 5 - 2y^2$.

2º) $- 4a^2b + 3xy^2 + 8 + 2a^2b - xy^2 - 4$.

3º) $3m - 4bm + 3b - 5m + 6bm + 7b - 5$.

Addition of algebraic expressions

To add algebraic expressions, we write them in succession by preserving the signs of their terms, and we reduce the like terms.

EXAMPLES

- Let $A = 3x^2 - 4x + 5$ and $B = - 5x^2 + 3x - 7$.

  $A + B = 3x^2 - 4x + 5 - 5x^2 + 3x - 7$

  $A + B = - 2x^2 - x - 2$.
- Let $P = 5b^2 + 7ab - 3a^2$ and $Q = 3b^2 - 2ab + c^2$.

  $P + Q = 5b^2 + 7ab - 3a^2 + 3b^2 - 2ab + c^2$

  $P + Q = 8b^2 + 5ab - 3a^2 + c^2$.

Application 6

Calculate the following algebraic expressions :

$C = 2x^2y - 5y^2 - 3x^2 + 2$ and $D = 4y^2 + 3x^2 - x^2y + 4$.

Subtraction of algebraic expressions

To subtract an algebraic expression from another, we write them in succession by changing the signs of the terms of the expression to be subtracted and we reduce the like terms.

EXAMPLES

- Let $A = 2x^3 - 4x + 5$ and $B = 2x - 7 + 5x^3$.

  $A - B = 2x^3 - 4x + 5 - 2x + 7 - 5x^3$

  $A - B = -\,3x^3 - 6x + 12$.

- Let $P = 3a^2 - 4ab + 5b$ and $Q = 2ab + a^2 - 5b + 8$.

  $P - Q = 3a^2 - 4ab + 5b - 2ab - a^2 + 5b - 8$

  $P - Q = 2a^2 - 6ab + 10b - 8$.

Application 7

Calculate $C - D$ where $C = 3xy^3 + 5x^2 - 8y + 7$ and $D = 2x^2 - xy^3 + 2y - 8$.

Exercises and problems

For testing the knowledge

1 Complete the following table :

monomial	variable	coefficient	exponent of the variable
$-\,6x^7$			
	t	$-\,3.54$	4
$5a^3$			
	z	6	2

2 Match each writing to its corresponding expression.

1°)

$1 \times x$ •	• $0.5x$
$0x$ •	• $2x$
$-1 \times x$ •	• 0
$\frac{1}{2}x$ •	• x
$x + x$ •	• $-x$

2°)

$x + y$ •	• The third of a number
$\frac{x}{2}$ •	• The quadruple of a number
$x + x$ •	• The sum of two numbers
$\frac{x}{4}$ •	• The quarter of a number
$4x$ •	• The double of a number
$2x$ •	• The triple of a number
$\frac{x}{3}$ •	• Half of a number
$3x$ •	• The reciprocal of a non-zero number
$-y$ •	• The opposite of a number
$\frac{1}{x}$ •	

3 Collect the like terms :

$4y^6$;	$2x^3$;	$-4a^2b^5$
$6a^2b^5$;	$2cza^2b^2$;	$-1.5a^2b^5$
$0.3x^3$;	$-5.6za^2b^2c$;	$3.5\ a^2b^5$

4 Reduce the like terms in each of the following algebraic expressions :

1°) $3x^5 - 8y^4 + 6a^2b - 7 + 2x^5 + 3y^4 - 2a^2b + 6$.

2°) $x^2y - 3y^2 + 5y^3x^2 - 3x^2y + 1.5y^2 - 0.2y^3x^2 - 4$.

3°) $4a^2bc^3 - 8t + 5a^2bc^3 - 4at + 10t - 5y + 2at$.

4°) $\frac{1}{3}x^2 + \frac{3}{5}x - \frac{1}{3}x + \frac{22}{29} + \frac{2}{3}x^2 - \frac{1}{15}x + \frac{4}{3}x + \frac{7}{29}$.

5 Calculate the numerical value of each of the following algebraic expressions .

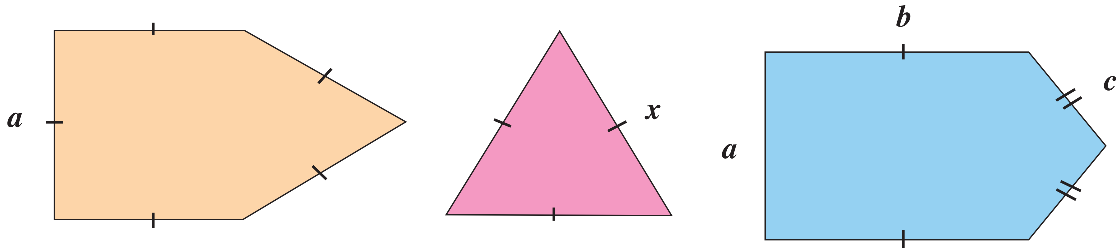
1°) $3a^3$ for $a = -2$.

2°) ab^2 for $a = -3$ and $b = 1$.

3°) $-3x^2y^3$ for $x = 2$ and $y = 1$.

4°) $-4x^2yz^3$ for $x = \frac{1}{2}$, $y = \frac{1}{3}$ and $z = -3$.

6 Write the algebraic expression for the perimeter of each of the figures below.



7 The side of a square is $3x$; calculate its perimeter.

8 Perform .

1°) $2a^3 \times 5a^2$.

2°) $5x^3 \times 2x$.

3°) $-\frac{1}{2}x^2 \times \frac{4}{3}x$.

4°) $y^3 \times (-\,2y^5)$.

5°) $\frac{3}{4}y \times \left(\frac{-8}{9}y^4\right)$.

6°) $-\,4x^2y^2 \times 3xy^2$.

9 Answer by true or false.

1°) Consider the algebraic expression:

$2x^3y - 6xy^5 + 4abx + 8$.

a) $2x^3y - 6xy^5$ is a monomial.

b) $4abx$ is a monomial.

2°) $3x^2 \times 5x^2 = 8x^2$.

3°) $3x^2 \times 5x^3 = 15x^5$.

4°) $4x^5 + 2x^5 = 6x^5$.

5°) $4x^5 + 2x^5 = 6x^{10}$.

6°) $2x^3 + 3x^2 = 5x^5$.

7°) $2a^3b + 3ab^3 = 6a^4b^4$.

For seeking

10 Given $A = 2x^3 - 4x^2 - 3x + 8$ and $B = x^4 - 2x^3 + 6x - 4$.

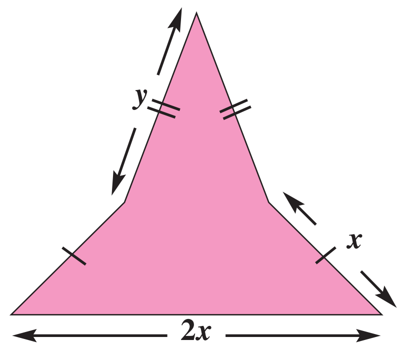
Calculate : $A + B$; $A - B$; $2A + B$; $3A - 2B$.

11 Given : $P = 3x^2y - 2xy^2 + 7xy - 3$ and $Q = -2x^2y - 6xy + 5 + 4xy^2$.

Calculate : $P + Q$; $P - Q$; $2P - 3Q$.

12 **1º)** Express the perimeter of the figure below in terms of x and y.

2º) Calculate the perimeter for $x = 5.5$ cm and $y = 6.3$ cm .



13 Perform .

1º) $-5x^5 \times \left(-\frac{1}{5}\ x^2\right)$

2º) $2ab^2 \times \left(-\frac{1}{2}\ a^2b\right)$

3º) $\frac{3}{5}\ xy^2 \times \left(-\frac{5}{3}\ x^2y^3\right)$

4º) $xy \times (-\ 2x^2y)$.

14 Given three similar monomials :

$A = \frac{3}{5}\ a^3b^2$, $B = -\frac{2}{3}\ a^3b^2$ and $C = -\ a^3b^2$

Calculate successively :

1º) $P = A + B - C$ **2º)** $Q = A - B + C$ **3º)** $R = -\ A + B + C$ **4º)** $S = A - B - C$.

15 Given the algebraic expressions:

$A = x^4 + 2x^3 - 5x^2 + 2x - 5$, $B = 2x^4 - 3x^2 + x + 3$, $C = 3x^4 + 2x^3 - x + 5$.

1°) Calculate successively :

a) $P = A + B - C$ **b)** $Q = A - B + C$ **c)** $R = B + C - A$

d) $S = P + Q + R$ **e)** $T = A + B + C$.

2°) Compare S and T.

TEST

1 Answer by true or false. **(6 points)**

1°) $72x$ is a monomial in the variable x.

2°) $13xy - 2x^2$ is an algebraic expression.

3°) 5 is the cœfficient of $5x^2$.

4°) $3x^4 \times 2x^5 = 6x^{20}$.

5°) $x - 1 - (x + y) + y$ is always equal to -1.

6°) The numerical value of the expression : $2 + 2x^3 - 2$ for $x = 2$ is 2^4.

2 Perform . **(2 points)**

1°) $\frac{3}{7}x^2 \times \left(-\frac{7}{3}x\right)$

2°) $5x^2y \times \frac{3}{5}xy$.

3 Reduce each of the following expressions, then calculate its numerical value for $x = 3$ and $y = 1$. **(2 points)**

1°) $2x - 4y + 8 - (x + y - 4)$.

2°) $(5x + 6y - 10) + (x - 9.2y + 7)$.

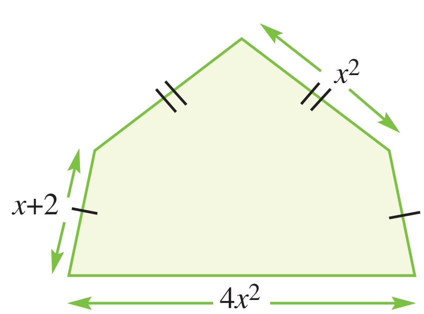
4 Given the algebraic expressions : **(6 points)**

$A = 3x^2 - 2x + 3$, $B = 2x^2 + 3x - 5$ and $C = x^2 + 5x - 8$.

Calculate : $R = A + B$; $S = A - B$ and $T = A + B - C$.

5 **1°)** Calculate the perimeter of the adjacent figure.

2°) Find this perimeter for $x = 2$.



(4 points)

16 EXPANDING - FACTORIZATION

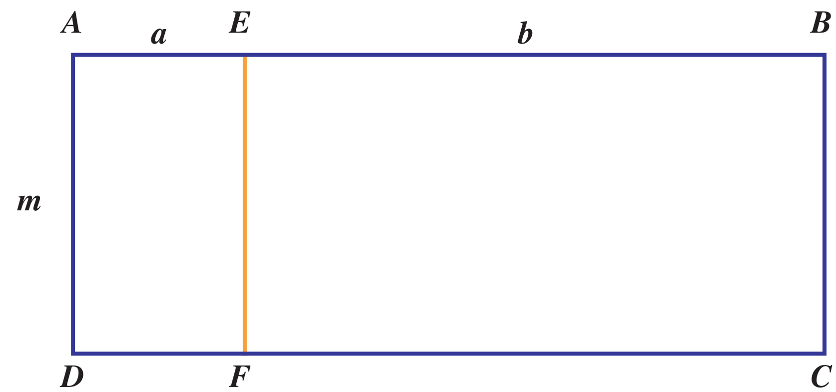
Objectives

- Developing and reducing algebraic expressions .
- Factorizing algebraic expressions .

CHAPTER PLAN

1 EXPANDING AND FACTORIZATION

Activity



Calculate the area of the rectangle *ABCD* in two ways :

1º) by calculating the product of its length by its width ;

2º) by calculating the sum of the areas of the two rectangles *AEFD* and *EBCF*. Which is the simpler way ?

Rules

• a , b and m being integers, we have :

$$\boxed{m\,(a + b) = ma + mb}$$

To expand the expression $m\,(a + b)$ is to replace it by $ma + mb$.

• a , b , m and n being integers, we have :

$$\boxed{(m + n)\,(a + b) = ma + mb + na + nb}$$

To expand the expression $(m + n)\,(a + b)$ is to replace it by

$ma + mb + na + nb$.

EXAMPLES

- $3\,(b + 2.5) = 3 \times b + 3 \times 2.5 = 3b + 7.5$.
- $(-2)\,(x + 5) = (-2) \times x + (-2) \times 5 = -2x - 10$.
- $(x + 5)\,(2x - 3) = 2x^2 - 3x + 10x - 15$
  $= 2x^2 + 7x - 15$.
- $3x\,(2x - 5) = 6x^2 - 15x$.
- $(3x + 5)\,(x^2 - 2x + 1) = 3x^3 - 6x^2 + 3x + 5x^2 - 10x + 5$
  $= 3x^3 - x^2 - 7x + 5$.

Application 1

Expand and reduce .

1º) $6\,(2 + y)$

2º) $-4\left(\frac{3}{2} - 2a\right)$

3º) $x\,(2 - y + a)$

4º) $(x - 1)\,(3x + 2)$.

2 FACTORIZATION

To factorize the expression $ma + mb$ is to replace it by $m(a + b)$.
***m* is a common factor of *ma* and *mb*.**

$$\boxed{ma + mb = m\,(a + b)}$$

EXAMPLES

- $\underline{5}\,x - \underline{5}\,y = \underline{5}\,(x - y)$;
- $7a - 7b + 14 = \underline{7}a - \underline{7}b + \underline{7} \times 2 = 7\,(a - b + 2)$;
- $4x^2 - 8x = \underline{4x} \times x - \underline{4x} \times 2 = \underline{4x}\,(x - 2)$;
- $2a\,(\underline{y - 1}) + 5b\,(\underline{y - 1}) = (\underline{y - 1})\,(2a + 5b)$.

Application 2

Factorize each of the following expressions :

$A = 4y + 8b + 16$

$B = 5y^2 - 10y$

$C = b\,(a^2 + 3) - 5\,(a^2 + 3)$

Remark :

The expanding and the factorization usually simplify the expression.

EXAMPLES

- $A = 34 \times 101 = 34\ (100 + 1) = 34 \times 100 + 34 \times 1 = 3\,400 + 34 = 3\,434.$
- $B = 13.8 \times 1.6 + 13.8 \times 8.4 = 13.8\ (1.6 + 8.4) = 13.8 \times 10 = 138$

Application 3

Calculate by expanding or by factorizing :

$A = 26 \times 12$ $\quad$ $B = 13 \times 99$ $\quad$ $C = 41 \times 11.2 - 41 \times 1.2$

$D = 69.1 \times 12 - 69.1 \times 2$ $\quad$ $E = 37.8 \times 7 + 37.8 \times 3$

EXERCISES AND PROBLEMS

For testing the knowledge

1 Expand.

1º) $5(a + b)$ **2º)** $-3\ (2a + 4b)$ **3º)** $2a\ (1 - b)$

4º) $m\ (-3 + m)$ **5º)** $-m\ (-3 + 4m)$ **6º)** $-2m\ (-m + n)$

2 Expand and reduce.

1º) $3\ (x - 1) - 5\ (x + 2) + 4x.$ **2º)** $3\ (x + y + 1) - 2\ (x - 2y) - 3y + 2.$

3º) $3\ (-2x + 5y + 4) - 2\ (-3x + 8y + 2) - y + 5.$ **4º)** $a\ (2 + a - b) - b\ (3 - a + b) + 4.$

3 Expand .

1°) $6(a+b)$. **2°)** $-2(c-5)$.

3°) $\left(\frac{3}{5}-2y\right)\times 4$. **4°)** $(2+a)(b-5)$.

5°) $(3-2y)(5+2a)$. **6°)** $(5-2y)(5-2y)$.

7°) $(-3a+2b)(-3a-2b)$.

8°) $(2z-5)(3z+6)$.

9°) $\left(\frac{2}{3}+2x\right)\left(-4x+\frac{8}{3}\right)$.

4 Factorize each of the following expressions .

1°) $9a+9b$ **2°)** $9a+18b$

3°) $16u-8$ **4°)** $4y^2-8xy$

5°) $7x+xy$ **6°)** $7x+14xy$

7°) $5x^2+15x$ **8°)** $4x^2y-16xy^2$

9°) $16ab-12ac$ **10°)** $14a-21$

11°) $3x^2-5x$ **12°)** $-9ab^2-6ab$

5 Factorize each of the following expressions .

1°) a^2+7a **2°)** $25a^2+30ab$

3°) $15a^2-10ab$ **4°)** $4b^2+2b$

5°) b^2-b **6°)** $4x^5-x^7$

7°) x^7-x^5 **8°)** $16a^4-8a^6$

9°) $21a^5-7a^6$ **10°)** $-10a^2b+5a^2x$

11°) $2xt+4xa-8xb$ **12°)** $14a^3b-7a^3b$

13°) $6ab-9ac-12at$

14°) $4x+8y+12z$

15°) $15ax-10ab+25bt$

6 Calculate in an easy way .

1°) $4\times(0.25-3)$

2°) $(60-2)\times 40$

3°) 176×101

4°) 787×99

7 Calculate in an easy way .

1°) $15.81\times 0.64+15.81\times 0.36$.

2°) $132.17\times 0.45+132.17\times 9.55$.

3°) $427.321\times 11.37-427.321\times 1.37$.

4°) $51.28\times 1.89-51.28\times 0.78-51.28\times 0.11$.

8 Verify the following equalities.

1°) $(x+y)(x+y)=(x+y)^2=x^2+2xy+y^2$.

2°) $(x-y)(x-y)=(x-y)^2=x^2-2xy+y^2$.

3°) $(x-y)(x+y)=x^2-y^2$.

9 Answer by true or false.

1º) $5(y-2) = 5y - 2.$

2º) $6(x+3) = 6x + 18.$

3º) $7(xy) = (7x)(7y).$

4º) $(a-2)(b-7) = ab + 14.$

5º) $(x+y) . (x+y) = x^2 + y^2.$

6º) $8x - 8y = 8(x-y).$

7º) $ax + x = (a+x)\,x.$

8º) $3(x+5) = 3x + 8.$

9º) $yx + zx = (y+z) . x.$

10º) $(x-2)(x+3)$ is a facctorized expression.

For seeking

10 Expand and reduce.

$A = 5(a+b-c) - 3(a-b-c-5)$

$B = 2x\,(x-1) + 3\,(y-2)$

$C = (y-7)\,(y-3)$

$D = (2b+1)\,(b-5)$

$E = (x-y)\,(y-2)$

$F = 2c\,(c-3) + (c-4)\,(c-1)$

$G = (x-3)\,(2x-1) + (3x-2)\,(3x+2)$

$H = (t+1)\,(2t-2) - (3-t)\,(3t-4) - t\,(t-7)$

$I = (2s+5)\,(2s-5) - s\,(s+3) + (s-2)\,(3s-1)$

11 Expand and reduce.

1º) $(x^3 - 2x^2)\,(x + x^2)$

2º) $(3xy - 1)\,(xy - 4)$

3º) $(a^2b - 3x^2)\,(2a^5 + 3x)$

4º) $(3a^5b^3 - b + 2a^7)\,(2a^2 - b^3)$

5º) $(6x^2 + 2xy - 5y^2)\,(2x^2 - xy + 3y^2)$

6º) $(4y^2 - 2x^2y^2 + 3)\,(-3y^2 + x^2y^2 - 1).$

12 Factorize each of the following expressions.

1º) $x(x+1) - 4\,(x+1)$

2º) $10a\,(x-5) - 15y\,(x-5)$

3º) $2y\,(a^2+1) - 5\,(a^2+1)$

4º) $(x+2) + 2x\,(x+2)$

5º) $4a(x-2) - 3b\,(x-2)$

6º) $4\,(x-3) - (x-3)$

13 Given $a = 125$ and $b = 225$.

1º) Write a and b as a product of prime factors.

2º) Calculate the G.C.F of a and b.

3º) Factorize the following expressions : $A = 225x - 125y$ and $B = 225x^2y - 125xy^2$.

14 Develop : $A = (x + 1)(2x - 3)(x + 4)$.

1º) Expand and reduce $(x + 1)(2x - 3)$.

2º) Reduce A.

3º) By using another way, expand and reduce A.

15 The given formula of the mass y (in kg) of an individual by using his height x (in cm) is :

$y = x - 100 - \dfrac{1}{4}(x - 150)$ for a man and $y = x - 100 - \dfrac{1}{2}(x - 150)$ for a woman.

1º) Reduce each of the two formulas.

2º) How much should a man weigh if his height is 180 cm?

Same question for a woman whose height is 160 cm.

16 The speed of swimming of a fish is given by this formula :

$V = \dfrac{L}{4}(1 + 3x) - \dfrac{5L}{4}$, where L is the length (in cm) of the fish, x is the number (per second) of the beating of its tail and V is the speed in cm per second.

1º) Reduce the expression of V.

2º) A red fish measures 20 cm. Its tail beats 1080 times per second. What is its speed in cm per second ? in m per second ?

TEST

1 **1º)** Expand and reduce the following expressions .

$A = 4\ (a^3 - 3a^2 + a) - 5\ (2a^3 - 4a^2 + 3a).$ **(1 point)**

$B = 6\ (a^4 - 2a^2 + 5) + 4\ (a^4 + 3a^2) - 3\ (a^4 - 2).$ **(1 point)**

2º) Calculate then A and B for $a = 1$ **(2 points)**

2 Calculate by expanding .

1º) 39×42 **2º)** 99×251 **(2 points)**

3 Expand and reduce the following expressions .

$A = (3a - 3)\ (3a + 2).$ **(1 point)**

$B = (x - 1)\ (2x + 3) - x + 7.$ **(1 point)**

$C = 2y\ (y - 4) + (y - 1)\ (y + 1).$ **(1 point)**

4 Factorize each of the following expressions.

$A = 16\ a^3 - 48a.$ **(1 point)**

$B = 5a\ (3x + 5) + 46\ (3x + 5).$ **(1 point)**

$C = (2x + 5)\ (3x + 7) - (2x + 5)\ .\ x.$ **(1 point)**

$D = 50\ a^3\ b^2 + 150\ a^4\ b^3 - 10\ ab^2.$ **(1 point)**

$E = 3x^3 - 2x^2 + 6x - 4.$ **(1 point)**

5 Factorize.

1º) $24y + 6$; **2º)** $z^2 - 5z$; **3º)** $12x^2 - 8x$.

4º) $5x^2 + 30$; **5º)** $42 - 14t$; **6º)** $y^2 - y$. **(3 points)**

6 **1º)** Find the G.C.F of 42 and 70. **(1 point)**

2º) Factorize the following expressions .

$A = 42x - 70y$. **(1 point)**

$B = 42\ ab^2 + 70ab$. **(1 point)**

17 CONGRUENT TRIANGLES (3)

Objective

Two triangles having their three sides respectively congruent are congruent .

CHAPTER PLAN

COURSE

1 - Third case of the congruency of triangles

2 - Commentary exercise

EXERCISES AND PROBLEMS

TEST

1 THIRD CASE OF THE CONGRUENCY OF TRIANGLES

Activity

1º) Draw a segment [*KL*] of measure 7 cm.
On the same side of [*KL*], draw an arc of a circle of center *K* and radius 6 cm, then an arc of a circle of center *L* and radius 4 cm. These two arcs intersect at *M*.
You constructed a triangle *KLM* knowing the measures of its three sides.

2º) Use the same procedure to construct a triangle *OPQ* such that :
$OP = 7$cm, $OQ = 6$cm and $PQ = 4$ cm.

3º) Copy each of these two triangles.

4º) Verify that these two copies are congruent.

5º) What are in these two triangles :

a) the congruent sides ?

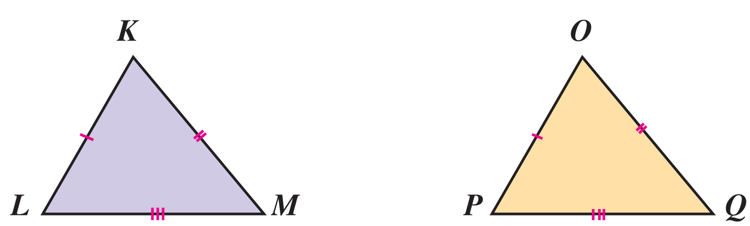
b) the equal angles ?

Rule

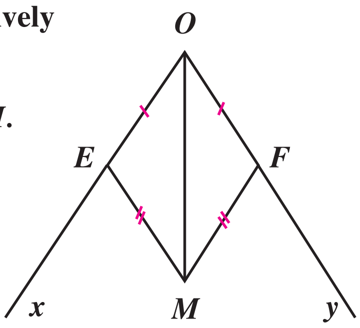
It the three sides of one triangle are respectively congruent to three sides of the other, then these triangles are congruent.

EXAMPLE

The two triangles below, *KLM* and *OPQ* are such that :
$KL = OP$, $KM = OQ$ and $LM = PQ$;
they are congruent (this is verified in the activity).



2 COMMENTARY EXERCISE



On the sides [Ox) and [Oy) of an angle $\widehat{xOy}$, we consider respectively the points E and F such that $OE = OF$.

M is a point in the interior of the angle $\widehat{xOy}$ such that : $EM = FM$.

Prove that [OM) is the bisector of angle $\widehat{xOy}$.

<u>Given :</u> $OE = OF$; $EM = FM$

<u>Required to prove :</u> $\widehat{EOM} = \widehat{FOM}$

Proof

Consider the two triangles OEM and OFM ; they have :

• $OE = OF$ (given) • $EM = FM$ (given) • [OM] common side.

These two triangles are congruent since the three sides of one triangle are respectively congruent to the three sides of the other.

All their corresponding parts are equal, in particular $\widehat{EOM} = \widehat{FOM}$, and hence [OM) is the bisector of angle $\widehat{xOy}$.

Application

[EI] is the median segment relative to the base [MN] of an isosceles triangle EMN.

a) Prove that [EI) is the bisector of angle $\widehat{MEN}$.

b) Prove that [EI] is the height relative to [MN].

EXERCISES AND PROBLEMS

For testing the knowledge

1 LMN and PQR are two triangles.
Verify whether the triangles are congruent in each of the following cases.

1°) $LM = PQ$, $MN = QR$, $LN = PR$.

2°) $LM = PQ$, $MN = QR$, $\widehat{LNM} = \widehat{PRQ}$.

3°) $MN = QR$, $\widehat{MLN} = \widehat{RPQ}$, $LN = PR$.

4°) $\widehat{MLN} = \widehat{RPQ}$, $\widehat{NML} = \widehat{PQR}$, $LM = QR$.

2 Given the two triangles LOI and RAT.
What should be added to the given equal parts so that these triangles become congruent ?
1º) $LO = RA$, $OI = AT$, **2º)** $LI = RT$, $\widehat{OLI} = \widehat{ART}$, **3º)** $\widehat{OIL} = \widehat{ATR} = 90°$, $OI = AT$.

3 Construct triangle MON such that : $MO = MN = 46$mm and $ON = 3$ cm.

4 Construct an equilateral triangle of perimeter 15 cm.

5 ABC and $A'B'C'$ are two triangles of perimeter 12 cm each.
Prove that they are congruent.

6 ABC is any triangle. [AM] is the median relative to [BC]. We produce [AM] to a length $ME = AM$.
1º) Prove that $BE = AC$.
2º) Prove that $AB = CE$.
3º) Prove that the two triangles ABC and BCE are congruent.

7 In triangle ABC, we produce the height [AH] to a length $HD = AH$.
1º) Prove that $AB = BD$.
2º) Prove that $AC = CD$.
3º) Prove that the two triangles ABC and DBC are congruent.

8 Two circles of centers O and I and radii r and r', respectively, intersect at V and R.
1º) Prove that the two triangles VOI and ROI are congruent.
2º) Deduce that [OI) is the bisector of $\widehat{VOR}$ and that [IO) is the bisector of $\widehat{VIR}$.

9 [AB] and [CD] are two congruent chords of a circle of center O.
Prove that $\widehat{AOB} = \widehat{COD}$.

10 Answer by true or false.

1º) Two triangles ABC and $A'B'C'$ are congruent if :

a) two sides of one are respectively congruent to two sides of the other,

b) $AB = A'C'$, $AC = A'B'$ and $\widehat{BAC} = \widehat{B'A'C'}$,

c) $AB = A'B'$ and $\widehat{ACB} = \widehat{A'C'B'}$,

d) $\widehat{ABC} = \widehat{A'B'C'}$, $\widehat{ACB} = \widehat{A'C'B'}$ and $\widehat{BAC} = \widehat{B'A'C'}$,

e) $AB = A'B'$, $AC = A'C'$ and $BC = B'C'$.

2º) In the two congruent triangles ABC and $A'B'C'$:

a) if $AB = A'B'$ then $\widehat{ACB} = \widehat{A'C'B'}$,

b) if $AC = A'C'$ and $AB = B'C'$ then $BC = A'B'$

For seeking

11 AEC is an isosceles triangle of vertex A. B is a point on $[AE]$ and D on $[AC]$ such that $AB = AD$.

1º) Prove that $CD = BE$.

2º) Prove that $DE = BC$.

3º) Prove that the two triangles DBE and DBC are congruent.

12 ABC is an isosceles triangle of vertex A. On the line (xy) holding $[BC]$, take the points D and E such that : $DB = BC = CE$.

1º) Prove that the two triangles ACD and ABE are congruent.

2º) Prove that the two triangles ABD and ACE are congruent.

13 SAC is an isosceles triangle of vertex S such that $SA = SC = 4$ cm. The sides $[AS]$ and $[CS]$ are extended on the same side of S of same length $SO = SI = 2$ cm (O is on (AS) and I is on (CS)).

1º) Prove that the two triangles SIA and SOC are congruent.

2º) Prove that the two triangles IAC and OAC are congruent.

14 Let $[Oz)$ be the bisector of $\widehat{xOy}$. M is a point of $[Oz)$; the perpendicular at M to $[Oz)$ cuts $[Ox)$ at A and $[Oy)$ at B.

1º) What is the nature of triangle AOB ?

2º) I is a point of $[Oz)$ such that $MI = MO$.

Prove that triangle OBI is isosceles. Deduce that $BI = AO$.

3º) P is the midpoint of $[OM]$ and J of $[MI]$.

Prove that $AP = BJ$.

4º) Prove that the two triangles OAP and BIJ are congruent.

15 A and D are two points on the perpendicular bisector of $[BC]$ and on the same side of $[BC]$.

1º) Prove that the two triangles ADB and ADC are congruent.

2º) (BD) cuts (AC) at E and (CD) cuts (AB) at F.

Prove that the two triangles BAE and CAF are congruent.

3º) Deduce that $DE = DF$ and that $AE = AF$.

What does (AD) represent to $[FE]$?

TEST

1 LMN and PQR are two triangles. Verify, in each of the following cases, whether the triangles are congruent.

1º) $MN = QR$, $\widehat{LMN} = \widehat{PQR}$, $\widehat{MLN} = \widehat{PRQ}$.

2º) $LN = PR$, $MN = QR$, $\widehat{LMN} = \widehat{PQR}$.

3º) $\widehat{MLN} = \widehat{QPR} = 90°$, $MN = QR$, $LM = PQ$.

4º) $LM = QR$, $MN = PR$, $LN = PQ$. **(4 points)**

2 Given the two triangles LOI and RAT . What should be added to the given equal parts so that the triangles become congruent?

1º) $IL = TR$, $\widehat{OLI} = \widehat{ART}$.

2º) $\widehat{LOI} = \widehat{RAT}$, $\widehat{OLI} = \widehat{ART}$. **(2 points)**

3 A, B and C are three points on a semi-circle of center O and radius R, such that $AB = BC$.

Prove that [OB) is the bisector of $\widehat{AOC}$. **(4 points)**

4 $\widehat{xOy}$ is any given angle. We take the points A and C on [Ox) and, B and D on [Oy) such that : $OA = OB$ and $OC = OD$.
[BC] and [AD] intersect at P .

1º) Prove that $AC = BD$.

2º) Prove the congruency of the triangles :

OAD and OBC ; ABD and ABC ; ADC and BDC ; PAC and PBD .

3º) Deduce that (PO) is the perpendicular bisector of [AB] and of [CD]. **(10 points)**

18 EQUATIONS

Objectives

- Adding and subtracting the same number from both sides of an equation does not change the equation.
- Replacing an equation by an equivalent equation.
- Reduce an equation to the form $ax = b$.
- The equation $ax = b$ has for solution $\frac{b}{a}$.
- Organizing the given and translating it into an equation of the form $ax = b$.

CHAPTER PLAN

COURSE

EXERCISES AND PROBLEMS

TEST

1 DEFINITION

The writing $2x - 3 = 5$ is called :
equation of the first degree in x, x is **the unknown** ; $2x - 3$ and 5 are the **sides** of this equation ; $2x$, -3 and 5 are the **terms**.

$x = 4$ verifies this equation since : $2 \times 4 - 3 = 8 - 3 = 5$.
4 is the **solution** or **the root** of $2x - 3 = 5$.
To solve an equation is to find the value of the unknown which verifies it.

Application 1

Consider the equation $3x + 1 = 4$.
Which of the following values is a solution of this equation ?

$x = 0$; $x = 1$; $x = 3$.

2 EQUIVALENT EQUATIONS

The equations $2 + x = 5$ and $4x = 12$ have the same solution $x = 3$.

They are called equivalent.

Two equations are said to be **equivalent** if they admit the **same solution**.

Application 2

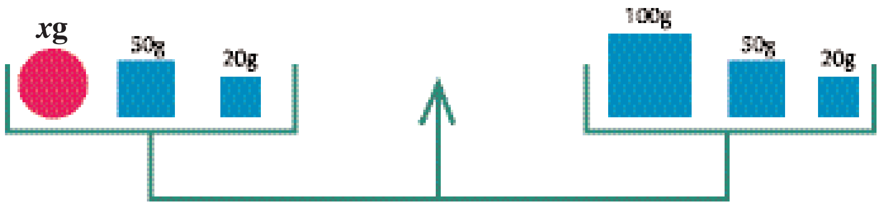
1º) Choose the correct answer.

The equation $2 + x = 6$ has for solution	$\boxed{0}$	$\boxed{4}$	$\boxed{1}$
The equation $x - 1 = -1$ has for solution	$\boxed{2}$	$\boxed{1}$	$\boxed{0}$
The equation $x - 3 = 1$ has for solution	$\boxed{4}$	$\boxed{3}$	$\boxed{2}$
The equation $x + 7 = 7$ has for solution	$\boxed{2}$	$\boxed{-1}$	$\boxed{0}$

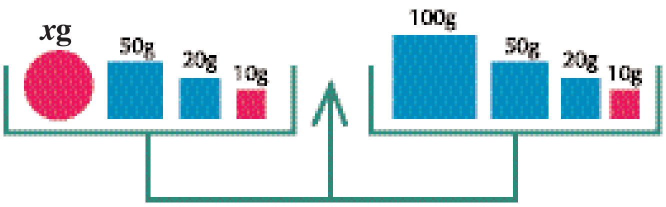
2º) Indicate which of the preceding equations are equivalent.

3 PROPERTIES AND SOLUTION

Activity

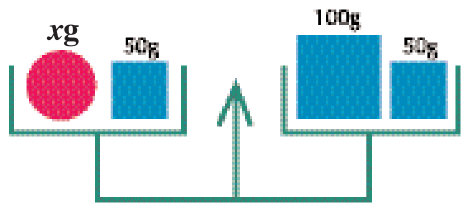


The balance is at equilibrium.
We have : $x + 50 + 20 = 100 + 50 + 20$



The balance remains at equilibrium.
Complete :

$x + 50 + 20 + ... = 100 + 50 + 20 + ...$



The balance remains at equilibrium.
Write the corresponding equation.

.... = ...

Property 1

If we add or subtract the same number from both sides of an equation, we still obtain an equation which has the same solution.

EXAMPLE

To solve the equation $5x - 2 = 4x + 4$.
Add 2 to both sides :

$5x - 2 \mathbf{+ 2} = 4x + 4 \mathbf{+ 2},$

then $5x = 4x + 6$.

Subtract $4x$ from both sides :

$5x \mathbf{- 4x} = 4x + 6 \mathbf{- 4x}.$

then $x = 6$.

These steps are summarized in the following manner :

$5x - 2 = 4x + 4$

$5x \mathbf{-} 4x = 4 \mathbf{+} 2$

$x = 6.$

6 is the solution of this equation.

By examining this work we derive the following rule.

In an equation we can transfer one term of one side to the other side on condition that we change the sign that precedes this term.

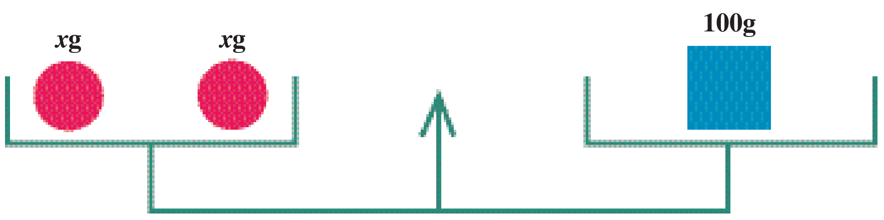
Application 3

Solve the following equations :

1º) $3x - 5 = 2x + 2$

2º) $2x - 8 = x + 3$.

Activity



The balance is at equilibrium.

• Write the corresponding equation.

• Can you deduce the value of x ?

If yes, complete $x =$ ....

Property 2

If we multiply or divide the two sides of an equation by the same number, we obtain an equation which has the same solution.

EXAMPLE

• The equation $4x - 5 = 2x + 3$ is written $4x - 2x = 5 + 3$

$2x = 8$

$\frac{2x}{\mathbf{2}} = \frac{8}{\mathbf{2}}$ and $x = 4$.

• The equation $\frac{x}{3} - 2 = 1$ is written $\frac{x}{3} = 2 + 1$; $\frac{x}{3} = 3$

$\frac{x}{3} \times \mathbf{3} = 3 \times \mathbf{3}$, then $x = 9$

Application 4

Solve each of the following equations :

1º) $7x - 1 = 2x + 4$

2º) $\frac{a}{5} - 4 = 1$.

General case: solution of the equation $ax = b$ with $a \neq 0$

By using the first property, every first degree equation can be written in the form $\boldsymbol{ax = b}$, where a and b are two numbers such that $a \neq 0$.
The second property gives :

$\frac{ax}{\boldsymbol{a}} = \frac{b}{\boldsymbol{a}}$, so $x = \frac{b}{a}$.

$$\boldsymbol{ax = b \text{ gives } x = \frac{b}{a}, \text{ where } a \neq 0}$$

Particular case

1º) The equation : $0x = b$ where $b \neq 0$ does not admit any solution,

2º) The equation : $0x = 0$ admits every number as solution.

Remark

When an equation admits denominators, we should :

• **Reduce all the terms to the same denominator,**

• **Remove this common denominator** : this is done by multiplying the two sides of the equation by the value of the denominator.

EXAMPLE

The equation $\frac{x}{2} + \frac{1}{6} = \frac{x}{3} + 2$ is written,

after being reduced to the same denominator 6:

$\frac{3x}{6} + \frac{1}{6} = \frac{2x}{6} + \frac{12}{6}$

or $3x + 1 = 2x + 12$, then $x = 11$.

Application 5

Solve each of the following equations :

1º) $\frac{x}{3} - 5 = \frac{1}{3} + \frac{x}{4}$

2º) $2b - \frac{1}{4} = \frac{b}{2}$

3º) $\frac{2x}{3} - 1 = 2\left(\frac{x}{3} - \frac{1}{2}\right)$

4º) $\frac{3y}{4} - \frac{1}{2} = 3\left(\frac{y}{4} + 1\right)$.

4 TRANSLATING INTO EQUATIONS

To solve a problem is to translate it into an equation, after a practical choice of the unknown for this. We follow these four steps :

1°) choice of the unknown (after the reading and the analysis of the text)
2°) Translating into an equation
3°) Solving the equation
4°) Checking by reading the given problem.

EXAMPLES

1°) In a school, the number of students in the three classes GR7, GR8 and GR9 is 95. The GR8 has 14 students less than GR7 and 3 students more than the GR9.
What is the number of students in each class ?

Let x be the number of students in GR8. The number of students of the GR7 is $x + 14$ and that of GR9 is $x - 3$.

Then we have :
$x + (x + 14) + (x - 3) = 95$, so $3x = 84$ then $x = 28$. The number of students of GR8 is 28, that of GR7 is 42 and that of GR9 is 25.

2°) The dimensions of a rectangle are 60 m and 45 m. We increase the length by 12m. How much should we decrease the width so that the area of the new rectangle is equal to the area of the original one ?

Let x be the amount to be decreased from the width.

The area of the first rectangle is :
$60 \times 45 = 2\,700$, so $2\,700$ m^2.

The area of the new rectangle is :
$(60 + 12)(45 - x) = 72(45 - x) = 3\,240 - 72x$, so $3\,240 - 72x$.

Since the two areas are equal, therefore :

$$2\,700 = 3\,240 - 72x,$$
$$72x = 3\,240 - 2\,700$$
$$72x = 540$$
$$x = \frac{540}{72} = 7.5 \text{ ; so } 7.5 \text{ m.}$$

The amount that should be decreased is 7.5 m.

Exercises and problems

For testing the knowledge

1 Verify whether each number is a solution of the equation :

1º) $3x - 7 = x + 5$ $\qquad x = 6.$

2º) $3x - 7 = 5x + 2$ $\qquad x = \dfrac{-9}{2}.$

3º) $2y - 7 = 8y + 2$ $\qquad y = 1.$

4º) $\dfrac{t-2}{3} = \dfrac{t-3}{2}$ $\qquad t = 5\ .$

5º) $7(b - 3) + 3\,(b + 12) = 9(b + 2)$ $\qquad b = 3.$

2 Solve the following equations : $2x = 4$; $5x = 0$; $-4x = 28$;

$3y - 7 = 2$; $\dfrac{-2b}{7} = 0$; $\dfrac{a}{3} = \dfrac{2}{3}$; $3x = 1$.

3 Fadi and Nadia solved the equation $\dfrac{x-2}{2} + \dfrac{x}{6} = \dfrac{5}{3}$ in the following manner :

Fadi

$$\frac{x-2}{2} + \frac{x}{6} = \frac{5}{3}$$

$$\frac{3(x-2)+x}{6} = \frac{10}{6}$$

$$3x - 6 + x = 10$$

$$4x = 10 + 6$$

$$4x = 16$$

$$x = \frac{16}{4}\ ,\ x = 4$$

Nadia

$$\frac{x-2}{2} + \frac{x}{6} = \frac{5}{3}$$

$$\frac{3(x-2)+x}{6} = \frac{10}{6}$$

$$3x - 6 + x = 10$$

$$4x = 10 - 6$$

$$4x = 4$$

$$x = \frac{4}{4}\ ,\ x = 1$$

Which solution is correct ? Justify you ranswer.

4 Solve each of the following equations :

1°) $5x + 7 = 4x + 8$ **2°)** $4 = 6 - 2a$ **3°)** $-7b = 2 - 5b$

4°) $4(3t + 2) = 3(t + 5)$ **5°)** $3(2m + 1) - 7 = 2m$ **6°)** $2\left(\frac{n}{5} + 1\right) = 0$

7°) $\frac{2y - 1}{5} = 2\left(\frac{y}{5} + 4\right)$ **8°)** $\frac{6 - z}{3} = 2$ **9°)** $\frac{x + 2}{2} - \frac{x + 9}{3} = -x$

10°) $\frac{1}{3}\left(\frac{x}{2} + 1\right) = \frac{1}{9}$ **11°)** $3(x - 1) - x = 2x - 3$ **12°)** $\frac{3x + 1}{4} - \frac{x + 3}{3} = \frac{1}{3} - x$.

5 For each of the problems below, designate a letter for the unknown, write an equation then solve it.

1°) The product of a number by 4.4 is 11. What is the number ?

2°) The product of 11 by a number is 4.4. What is the number ?

3°) The third of a number is 11. What is the number ?

4°) The sum of a number and 4.4 is 11. What is the number ?

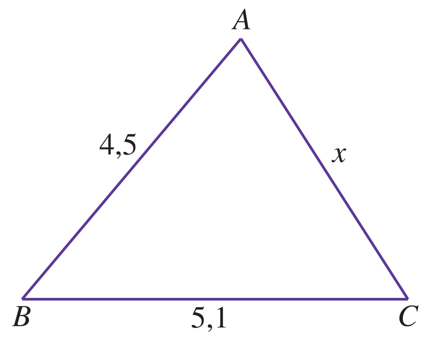
5°) The sum of 11 and a number is 4.4. What is the number ?

6°) The quotient of a number by 11 is 4.4. What is the number ?

7°) The double of a number is 11. What is the number ?

8°) The sum of 4.4 and triple a number is 11. What is the number ?

6 The perimeter of the triangle below measures 12.8 cm.



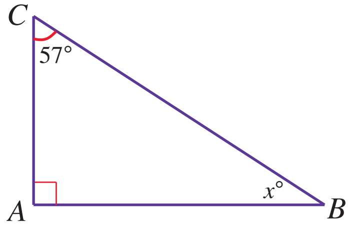
1°) Which equation helps us to find x ?

a) $x + 4.5 = 5.1 + 12.8$ **b)** $x - 4.5 = 12.8 - 5.1$

c) $x \times 5.1 + 4.5 = 12.8$ **d)** $x + 4.5 + 5.1 = 12.8$

2°) Find the value of x .

7 Write an equation in x that translates the given situation, then solve this equation.



8 **1º)** Given two consecutive numbers. If x is the first number express the second number in terms of x .

2º) The sum of these two numbers is 35. Find these numbers.

9 The length of the rectangle below is double its width.

1º) Express the length in terms of x.
2º) The perimeter of this rectangle is 30cm. Find its dimensions.

10 In a problem, Sami wrote the following equation :
$5x - 18 = 6x - 8$ in order to find the price x of a pen.

1º) Solve this equation. **2º)** Why isn't this equation correct ?

11 Answer by true or false.

1º) $2x - 1 = x + 4$ has a solution $x = 5$. **2º)** $\frac{3x}{5} = 0$ has a solution $x = \frac{5}{3}$.

3º) The two equations $2x - 8 = 0$ and $x + 2 = 6$ are equivalent.
4º) The equation $2x + 1 = 2(x + 3)$ admits no solution.
5º) Every number is a solution of the equation $3x - 4 = x + 2(x - 2)$.

6º) $9x = 9$ gives $x = 0$. **7º)** $3x = 5$ gives $x = \frac{3}{5}$.

8º) $z - 6 = 5$ gives $z = \frac{5}{6}$. **9º)** $4y = 6$ gives $y = 2$.

10º) $5a = \frac{2}{5}$ gives $a = 2$.

For seeking

12 Solve the following equations :

1°) $\frac{x-1}{3} - \frac{2x-3}{2} = \frac{x}{6} - \frac{x+1}{3}$

2°) $3\left(x - \frac{3}{2}\right) - \frac{x+4}{3} = x - 1$

3°) $\frac{x-2}{5} - \frac{x+2}{3} = -\frac{2x}{15}$

4°) $\frac{x-2}{5} + \frac{1}{3} = \frac{x}{5} - \frac{1}{15}$.

13 A basket of apples and a basket of grapes both weigh 20 kg. The first basket weighs 8 kg more than the second. How much does each basket weigh ?

14 The sum of two numbers is 400. Calculate these two numbers knowing that one of them is the triple of the other.

15 If we add 25 to a given number and subtract 12 from the obtained result, we obtain 16. What is the number ?

16 Nabil asked Diana to choose a number. He gave her the following hint :

- add 7
- multiply by 2 the obtained result
- subtract 4.

Diana answered «I obtain 20».

What number did Diana choose ?

17 The difference between two numbers is 42. Find these numbers knowing that the greater is the quadruple of the smaller.

18 The sum of three consecutive numbers is 102. Find these numbers.

19 If I add 4 to the triple of the grade taken by Karim on his mathematics homework I get 40. What is his grade?

20 How can we divide the sum 875 000L.L. between two people such that one of them has 15 000L.L. more than the other ?

21 Chadi wants to buy a C.D album which costs 63 000L.L. He does not have enough money. His brother Ziad gave him 8 000 L.L. Chadi buys the album and has no money left. How much money did Chadi have ?

22 The price of a cinema ticket is 10000L.L. for an adult and 7000L.L. for a child. 60 people attended the movie and paid 540 000L.L.
Find the number of children and adults that attended the movie.

23 Sami is 4 years older than Ziad. Chadi is as old as Sami and Ziad together. The sum of their ages is 48 years. What is the age of each ?

24 The age of a father is 34 years, and the ages of his two sons, Karim and Walid, are 12 years and 8 years respectively. In how many years will the sum of the ages of the sons be equal to the age of the father ?

25 A father is 24 years older than his son. What are their ages knowing that, in ten years, the sum of their ages will be 68 years ?

26 **1°)** Recopy and complete the following table .

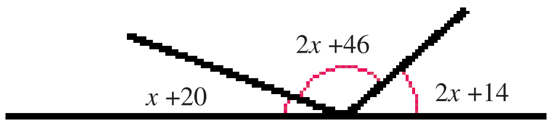
	Hadi	**Lama**
Actual age	15	x
Age after 10 years		

2°) Let $A = 25 + (x + 10)$. What does A represent ?
3°) Calculate Lama's age if $A = 45$.

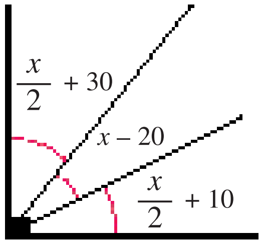
27 A sum of money is distributed among a number of children. If each child was given 28 000 L.L., the remaining amount is 30 000L.L. If 29 000L.L. was given to each, the remaining amount is 15 000L.L. Find the number of children and the distributed amount.

28 Calculate x in each of the following cases (the angles are expressed in degrees).

1º)

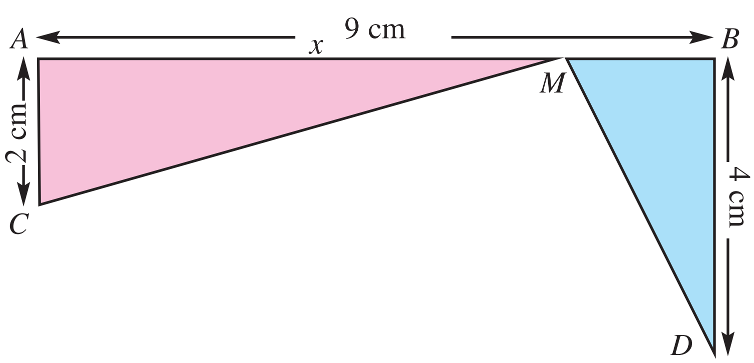


2º)

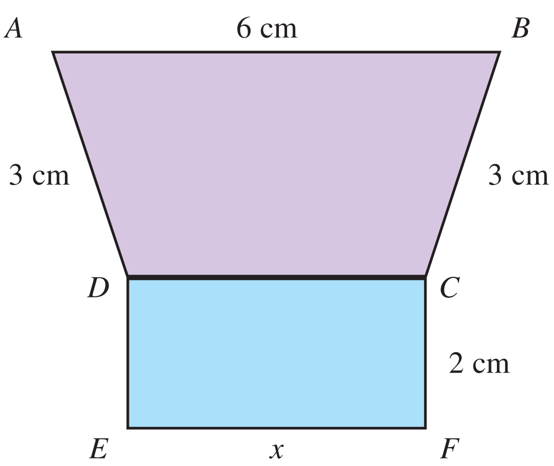


29 Find the dimensions of a rectangle knowing that its perimeter is 372 m and that the length is 15 m less than the double of the width.

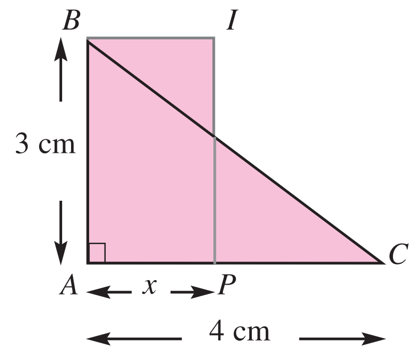
30 Locate M on $[AB]$, so that the two triangles AMC and BMD would have the same area.



31 Calculate x such that the perimeter of the rectangle $CDEF$ is equal to half of that of the isosceles trapezoid $ABCD$.



32 In the adjacent figure, P varies between A and C .



1º) How does x vary ?

2º) Calculate x so that the area of the rectangle *ABIP* is equal to that of triangle *ABC*.

3º) What is then the position of P?

33 A shopkeeper sells the third of the eggs he has in his basket. He broke 3 eggs by accident and he still has $\frac{5}{8}$ of the basket. How many eggs were in the basket ?

34 Two pieces of material measure 120 m and 46m each. We cut from each piece the same number of meters. The first piece is then three times longer then the second.
How many meters were cut from each piece ?

35 Two cyclists start at the same time,

one from Beirut (B), the other from Tripoli (T), and go to the meeting point.
The speed of the first is 16 km per hour, and that of the second is 24 km per hour.
At what distance from Beirut will they meet ?

Indication :

- the distance between the cities is 80 km.
- the distance = the time × the speed.

TEST

1 Choose the equation which translates the given situation.

a) The half of a number is obtained by subtracting 48 from its double. **(2 points)**

$\frac{x}{2} = \frac{2x - 48}{2}$ ☐ $\quad \frac{x}{2} = 2x - 48$ ☐ $\quad \frac{x}{2} = 48 - 2x$ ☐

b) The age of a father is triple that of his son.
In ten years the age of the father will be double the age of his son. **(2 points)**

$3x = 2x + 10$ ☐ $\quad 3x + 10 = 2x$ ☐ $\quad 3x + 10 = 2(x + 10)$ ☐

c) Walid spent 10,000L.L. in a shop, that is 1000L.L. more than half of the amount he had initially. **(2 points)**

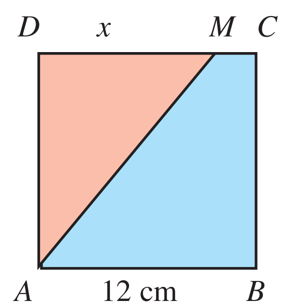
$x - 10\,000 = \frac{x}{2} + 1000$ ☐ $\quad \frac{x}{2} + 1000 = 10\,000$ ☐ $\quad x - 1000 = \frac{10\,000}{2}$ ☐

2 Solve each of the following equations : **(6 points)**

a) $3(x - \frac{1}{4}) - \frac{3}{2}x = x - 1$ **b)** $\frac{2\,(20x - 1)}{3} = 10x + 1$

c) $\frac{5x - 2}{4} - \frac{7x - 3}{8} = \frac{x - 1}{2}$

3 Calculate x so that the area of triangle AMD is one third the area of square $ABCD$. **(2 points)**



4 A math teacher gives his student 16 problems. He gave him 5 points for each correct exercise and removed 3 points for each wrong exercise. In the end the student had no points at all.

Find the number of the correct exercises. **(3 points)**

5 Two cyclists start from the same city and follow the same road. The speed of the first is 3 km per hour more than the second. The first cycled for 4 hours and the second for 3 hours. They reach two villages, respectively, separated by a distance of 30km.

What is the speed of each one ? **(3 points)**

19 PARALLEL STRAIGHT LINES (1)

Objectives

- Using the properties concerning parallel and perpendicular straight lines .
- Using Euclid's postulate in proofs.

CHAPTER PLAN

COURSE

1 - Parallel straight lines
  a) Definition
  b) Euclid's postulate
  c) Properties

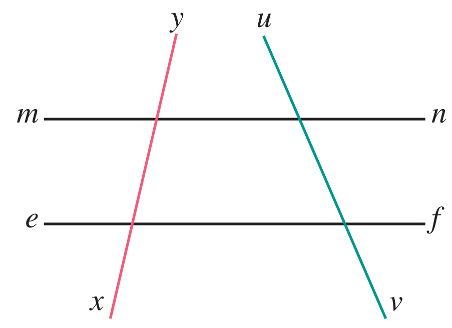
2 - Perpendicular drawn from a point to a straight line
  a) Definition
  b) Properties
  c) Construction

EXERCISES AND PROBLEMS

TEST

1 PARALLEL STRAIGHT LINES

Activity



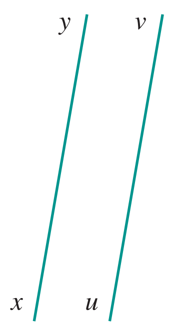
Observe the adjacent figure.

Does (uv) cut (mn) ?

Does (xy) cut (ef) ?

Is it the same for (uv) and (xy) ? For (mn) and (ef) ?

Definition

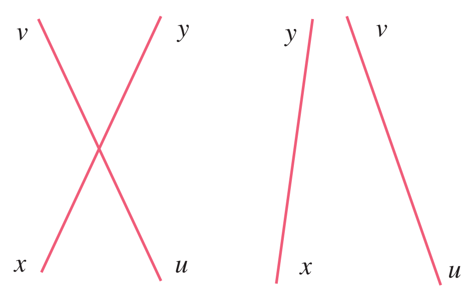


In a plane, two straight lines are parallel if they do not intersect.

(xy) and (uv) do not intersect, so they are parallel.

We write : **(xy) // (uv)** and we read : (xy) is parallel to (uv).

We also say that (xy) and (uv) have the same direction.

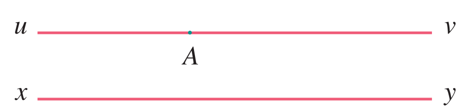


(xy) and (uv) are not parallel, they are **secant** (intersecting lines).

Euclid's postulate

From a given point not on a straight line, we can draw one and only one line parallel to the straight line.

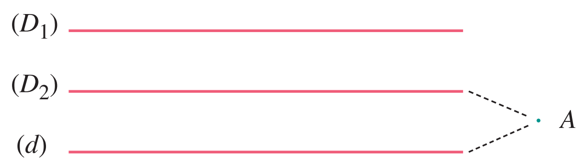
Observe the adjacent figure.



(uv) is the only straight line drawn from A and parallel to (xy).

2 PROPERTIES

1) In the figure below, (D_1) is parallel to (D_2) and (d) is parallel to (D_1).



Then (d) is parallel to (D_2) since :
If (d) and (D_2) intersect at A, for example, we will have from A two parallels to (D_1), which contradicts Euclid's postulate.

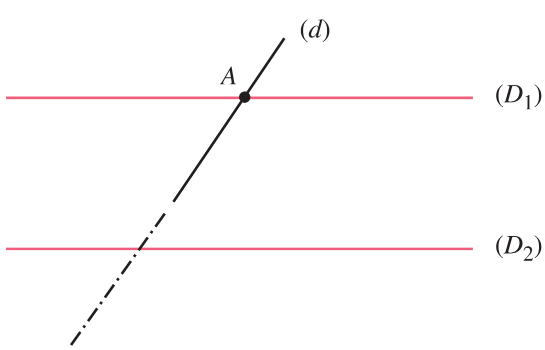
Hence the property :

if two straight lines are parallel, then every straight line parallel to one of them is parallel to the other.

We can also say :

two straight lines parallel to a third are parallel to each other.

2) In the figure below, (D_1) and (D_2) are parallel and (d) cuts (D_1) at A .



Then (d) cuts also (D_2) since :
if (d) is parallel to (D_2), we will have from A two parallels (D_1) and (d) to (D_2), which contradicts Euclid's postulate.

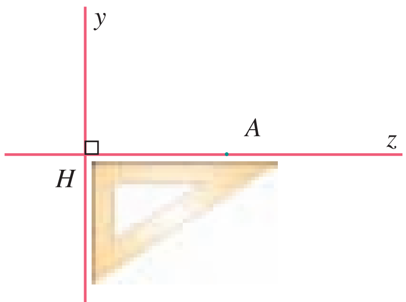
Hence the property :

if two lines are parallel, then every straight line cutting one of them cuts the other.

3 PERPENDICULAR DRAWN FROM A POINT TO A STRAIGHT LINE

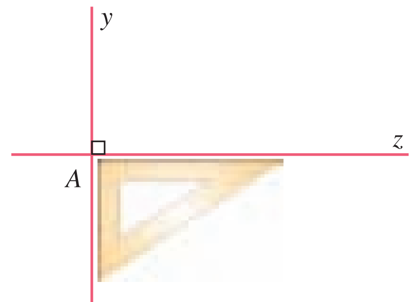
Definition

In the figures below :



A does not belong to (*xy*)

the straight line (*Az*) is perpendicular to (*xy*) at *H*.
We write : **(*Az*) ⊥ (*xy*).**
The distance from *A* to (*xy*) is *AH*.



A belongs to (*xy*)

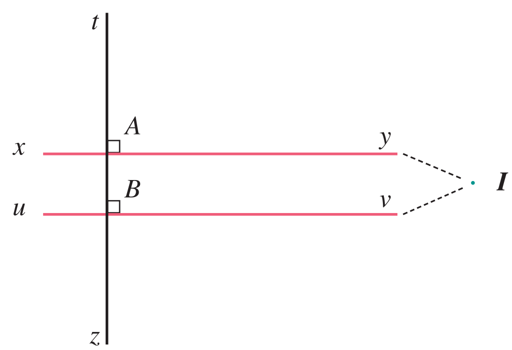
the straight line (*Az*) is perpendicular to (*xy*) at *A*.
We write : **(*Az*) ⊥ (*xy*).**
The distance from *A* to (*xy*) is zero.

Properties

1°) **From a point, we can draw one and only one perpendicular to a straight line.**

2°) In the adjacent figure, (*xy*) and (*uv*) are perpendicular to (*zt*).

Then (*xy*) is parallel to (*uv*) since, if (*xy*) and (*uv*) intersect at *I* for example, we will have from *I* two perpendiculars to the same straight line (*zt*) which is impossible.



Hence the property :

two straight lines perpendicular to a third are parallel.

The distance between the two parallel straight lines (xy) and (uv) is AB .

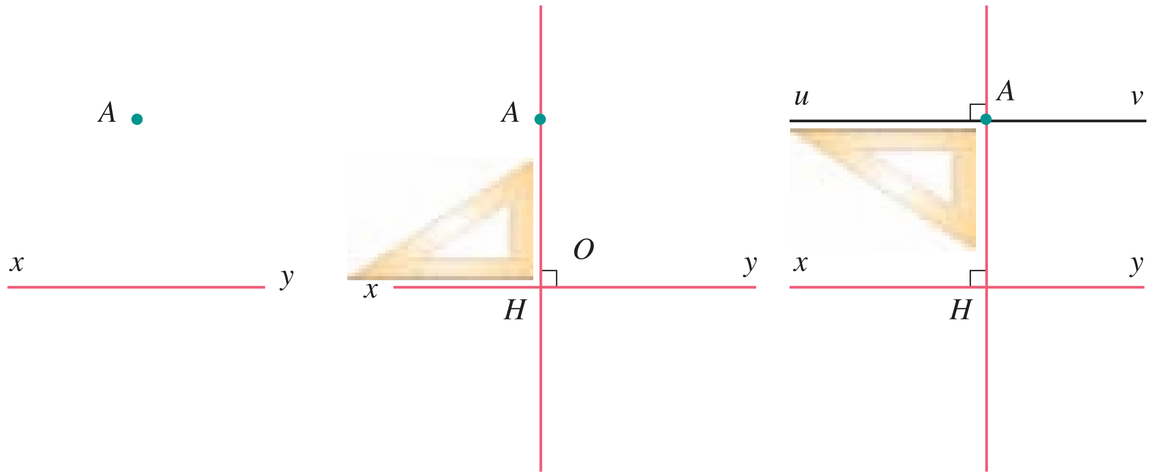
(xy) is parallel to (uv) and (zt) is perpendicular to (xy) .

Therefore (zt) is perpendicular to (uv) .

3º) If two straight lines are parallel, every perpendicular to one of them is also perpendicular to the other.

Construction

Constructing the parallel to (xy) passing through A.



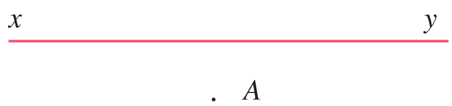
- We draw the straight line passing through A and perpendicular to (xy) at H.
- We draw the straight line (uv) passing through A and perpendicular to (AH).
- The two straight lines (uv) and (xy), being perpendicular to the same straight line (AH), are parallel.

  (uv) is then the required straight line.

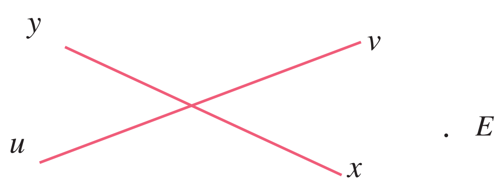
EXERCISES AND PROBLEMS

For testing the knowledge

1 A is a point not on the straight line (xy).
Draw from A, the parallel to (xy).



2 Draw from E the straight line (zt) parallel to (xy).
Does (zt) cut (uv)? Why?



3 ABC is any triangle. Draw :
(Ax) perpendicular to (BC), (By) perpendicular to (BC) and (Cz) parallel to (Ax).

1°) Justify the following :

a) (Ax) // (By).
b) (Cz) // (By).
c) $(Cz) \perp (BC)$.
d) (AB) cuts (Cz).
e) (AC) cuts (By).

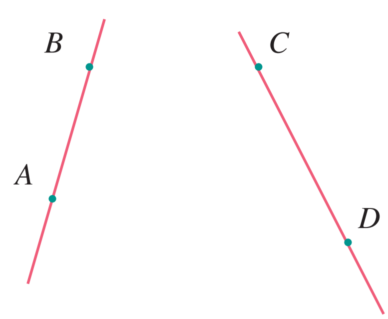
2°) Let $[AM]$ be the median relative to $[BC]$.
Does (AM) cut (By)? Why?

4 ABC is any given triangle.
Draw the straight lines passing through A, B and C and parallel to the straight lines (BC), (AC) and (AB) respectively.

5 **1°)** Draw a figure using the following indications :
(xy) // (uv) ; $(uv) \perp (ir)$;
$(mn) \perp (xy)$; (mn) // (pq) ;
$(zt) \perp (pq)$.

2°) List other parallel straight lines and perpendicular lines of the figure and justify.

6 Do the lines (AB) and (CD) of the figure below have a common point ?
Are they parallel ?
Do segments $[AB]$ and $[CD]$ have a common point ?



7 Answer by true or false.

1º) If two straight lines (D_2) and (D_3) are perpendicular to a same straight line (D_1), then (D_2) and (D_3) are perpendicular.

2º) If two straight lines are parallel, every straight line perpendicular to one of them is perpendicular to the other.

3º) From a point not on a straight line, we can draw many parallels to this straight line.

4º) Two straight lines perpendicular to a third are parallel.

5º) If two straight lines are parallel, every straight line which cuts one of them is parallel to the other.

6º) If two straight lines (AB) and (AC) have the same direction, then the three points A, B and C are collinear.

7º) From a given point, we can draw many perpendiculars to a given straight line.

8º) Two straight lines (Ax) and (Az) perpendicular to the same straight line (uv) are confounded.

9º) (D) and (D') are two perpendicular straight lines.
Every parallel line to one of them is perpendicular to the other.

For seeking

8 (xy) is a given straight line and point A is at a distance of 3 cm from it.

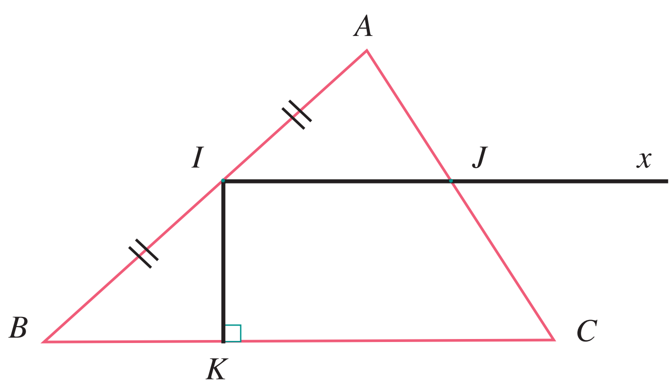
1º) Draw the straight line (uv) passing through A and parallel to (xy). What is the distance between the two parallel lines (xy) and (uv)?

2º) Can you find a line (zt) parallel to (xy) and at 3 cm from it (xy)?

9 **1º)** Draw two parallel straight lines (D_1) and (D_2) at a distance of 4 cm from each other.

2º) Draw the straight line (d) equidistant from (D_1) and (D_2).

10 In the adjacent figure, (Ix) is parallel to (BC).
What indications can you give a student who does not see this figure so that he will be able to draw it ?



11 Given two parallel straight lines (xy) and (uv). (zt) is perpendicular to (xy) and (uv), and cuts them at A and B respectively. From the midpoint O of $[AB]$, we draw a straight line which cuts (xy) and (uv) at M and N respectively.

1º) Prove that $\widehat{AMN} = \widehat{BNM}$.

2º) List the equal acute angles and the equal obtuse angles of the figure.

TEST

1 B is a point taken in the interior of a right angle $\widehat{xAy}$. The parallels drawn from B to (Ax) and to (Ay), cut (Ay) and (Ax) at E and F respectively.

Prove that the quadrilateral $AEBF$ is a rectangle. **(2 points)**

2 $ABCD$ is a rectangle.

1º) Which segments are held by parallel lines ? Why ? **(2 points)**

2º) $[AC]$ and $[BD]$ intersect at O. E and F are the feet of the perpendiculars drawn from O to $[AD]$ and $[BC]$ respectively.

Prove that the two straight lines (OE) and (OF) are confounded. **(4 points)**

3 $\widehat{xOy}$ and $\widehat{yOz}$ are two adjacent supplementary angles such that $\widehat{xOy} = 60°$.

$[Ou)$ and $[Ov)$ are the bisectors of $\widehat{xOy}$ and $\widehat{yOz}$ respectively. M is any point of $[Oy)$, P is the feet of the perpendicular drawn from M to $[Ov)$.

1º) Calculate $\widehat{yOz}$ and $\widehat{uOv}$. **(2 points)**

2º) Prove that (MP) is parallel to (Ou). **(4 points)**

4 ABC is any triangle. E is a point of (AC) such that $AE = AB$. The bisector of $\widehat{BAC}$ cuts $[BC]$ at M. (xy) is a straight line perpendicular to (AM) at A.

1º) What is (AM) in triangle ABE ? **(2 points)**

2º) Prove that (xy) is parallel to (BE). **(2 points)**

20 PARALLEL STRAIGHT LINES (2)

Objectives

- Identifying the alternate interior angles and the corresponding angles formed by two straight lines and cut by a transversal.
- Knowing the property that the alternate interior angles formed by two parallel straight lines and a transversal are equal.
- Knowing the property that the corresponding angles formed by two parallel straight lines and a transversal are equal.
- Knowing the converses of the two previous properties.
- Knowing that the sum of the angles in a triangle is 180° .

CHAPTER PLAN

COURSE

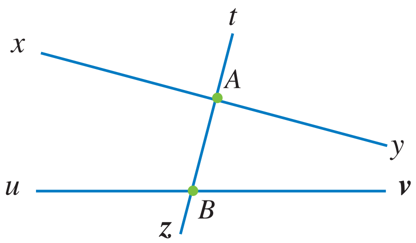
1. Definitions
2. Properties
3. Commentary exercise

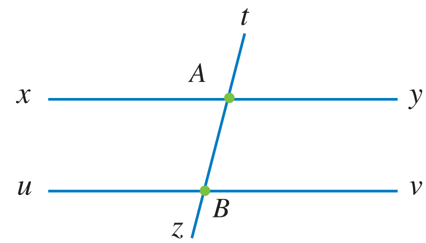
EXERCISES AND PROBLEMS

TEST

1 DEFINITIONS

In the figures below :





1°) a) $\widehat{xAB}$, $\widehat{yAB}$, $\widehat{uBA}$ and $\widehat{vBA}$ are called **interior angles** to the straight lines (xy) and (uv).

b) $\widehat{xAt}$, $\widehat{yAt}$, $\widehat{uBz}$ and $\widehat{vBz}$ are called **exterior angles** to the straight lines (xy) and (uv).

2°) $\widehat{xAB}$ and $\widehat{vBA}$ are called **alternate interior angles**. The same for $\widehat{yAB}$ and $\widehat{uBA}$.

3°) $\widehat{xAt}$ and $\widehat{uBA}$ are called **corresponding angles.**

The same for : $\widehat{yAt}$ and $\widehat{vBA}$; $\widehat{xAB}$ and $\widehat{uBz}$; $\widehat{yAB}$ and $\widehat{vBz}$.

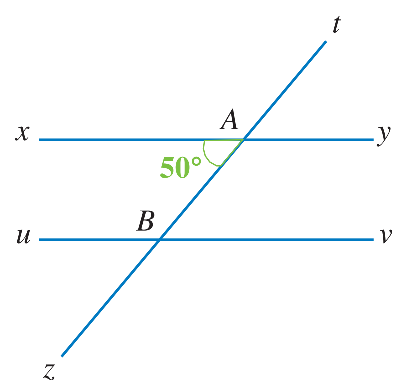
2 PROPERTIES

A Activity

(xy) and (uv) are two parallel straight lines.
A is a point of (xy), (zt) a transversal passing through A such that $\widehat{xAz} = 50°$ and which cuts (uv) at B.

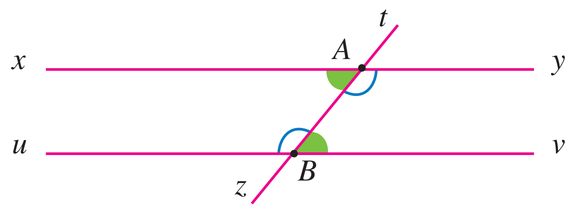
1°) Measure angles $\widehat{vBA}$ and $\widehat{uBz}$.

2°) Verify that $\widehat{xAB} = \widehat{vBA}$ and $\widehat{xAB} = \widehat{uBz}$.



Rule

(xy) and (uv) are two parallel straight lines cut by a transversal (zt).

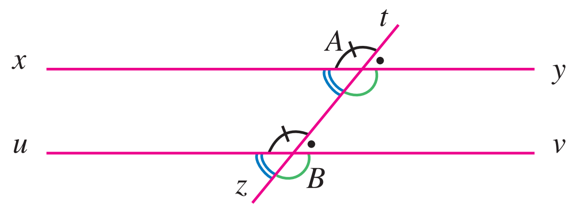


1º) **Two alternate interior angles are equal**.

$\widehat{xAB} = \widehat{vBA}$ and $\widehat{yAB} = \widehat{uBA}$ *(activity)*.

2º) **Two corresponding angles are equal.**

$\widehat{xAB} = \widehat{uBz}$; $\widehat{yAB} = \widehat{zBv}$; $\widehat{yAt} = \widehat{vBA}$; $\widehat{xAt} = \widehat{uBA}$.



B Activity

1º) Draw an angle $\widehat{xAy}$ of measure 40°.

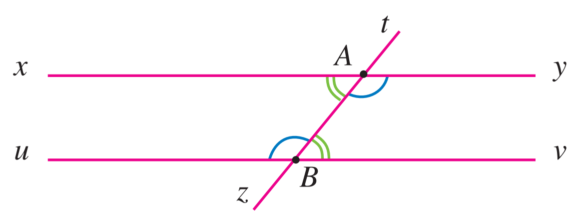
2º) From a point B of $[Ay)$, draw $[Bu)$ such that $\widehat{ABu} = 40°$ where $[Ax)$ and $[Bu)$ are on opposite sides of (AB).

3º) Are the straight lines (Ax) and (Bu) parallel ?

Rule

(xy) and (uv) are two straight lines cut by a transversal (zt).

If two alternate interior angles are equal then (xy) is parallel to (uv).



For example : if $\widehat{xAB} = \widehat{vBA}$ then $(xy) \mathbin{/\!/} (uv)$.

C Activity

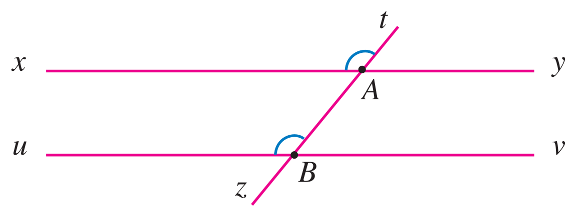
1º) Draw an angle $\widehat{xAy}$ of measure 70°.

2º) From a point B of $[Ay)$, draw $[Bu)$ such that $\widehat{yBu} = 70°$ with $[Ax)$ and $[Bu)$ being on the same side of (AB).

3º) Are the straight lines (Ax) and (Bu) parallel?

Rule

If two corresponding angles are equal then (xy) is parallel to (uv).



For example : if $\widehat{xAt} = \widehat{uBA}$ then $(xy) /\!/ (uv)$.

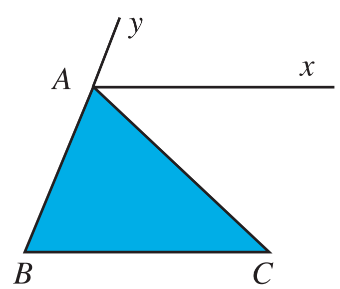
3 COMMENTARY EXERCISE

ABC is any triangle and (Ax) is parallel to (BC).

Prove that :

$\widehat{BAC} + \widehat{ABC} + \widehat{BCA} = 180°$**.**

Proof



We have : $\widehat{ABC} = \widehat{yAx}$ (corresponding angles)

and $\widehat{ACB} = \widehat{CAx}$ (alternate interior angles).

But : $\widehat{yAx} + \widehat{CAx} + \widehat{CAB} = 180°$ (straight angle).

Therefore : $\widehat{ABC} + \widehat{ACB} + \widehat{CAB} = 180°$.

The sum of the angles in a triangle is equal 180° .

Exercises and problems

For testing the knowledge

1 $[AB]$ and $[CD]$ are two segments intersecting at their midpoint O.

1º) Prove that the two triangles OAC and OBD are congruent ; deduce that (BD) is parallel to (AC).

2º) Prove that (AD) is parallel to (BC).

2 $[Ou)$ is the bisector of angle $\widehat{xOy}$, B is a point of $[Ox)$. The parallel to $[Oy)$ through B cuts $[Ou)$ at N.

Show that triangle BON is isosceles.

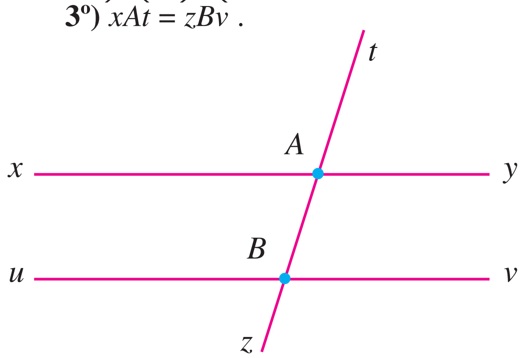
3 In the figure below, (xy) and (uv) are two parallel straight lines cut by the transversal (zt) at A and B respectively.

Prove that :

1º) $\widehat{xAB} + \widehat{uBA} = 180°$

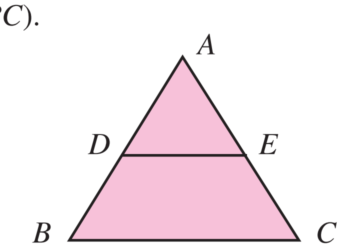
2º) $\widehat{yAB} + \widehat{vBA} = 180°$

3º) $\widehat{xAt} = \widehat{zBv}$.



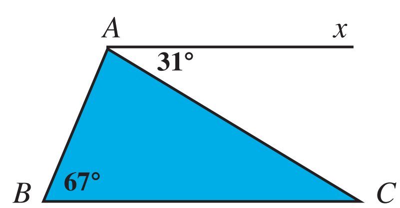
4 Observe the given figures and answer the following questions.

1º)
$AB = AC$ and $(DE) /\!/ (BC)$.



Show that triangle ADE is isosceles.

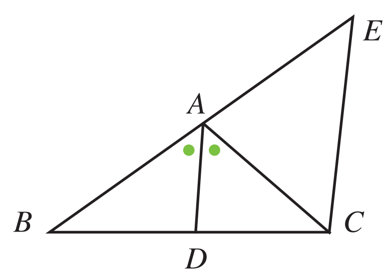
2º)



$(Ax) /\!/ (BC)$.

Calculate $\widehat{BAC}$.

3º) $[AD)$ is the bisector of $\widehat{BAC}$ and $(AD) /\!/ (CE)$.

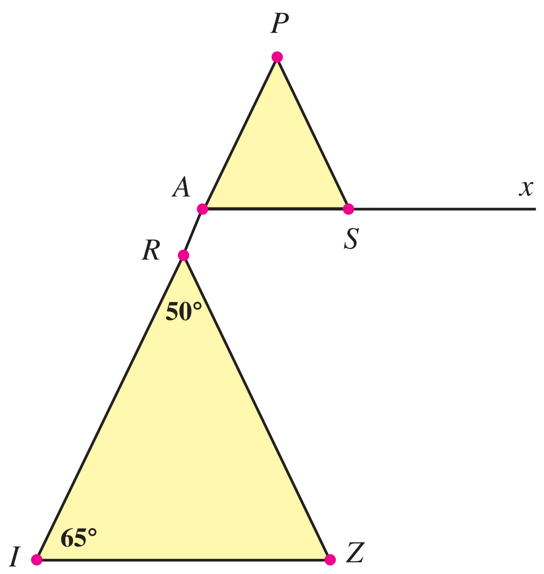


Prove that triangle ACE is isosceles.

5 In the figure below, $(AS) \parallel (IZ)$ and $(PS) \parallel (RZ)$.

1°) Prove that the triangle PAS is isosceles.

2°) Calculate $\widehat{PSx}$.



6 ABC is an isosceles triangle of vertex A and such that $\widehat{BAC} = 50°$.
Calculate the measure of the base angles of this triangle.

7 MNP is triangle such that :
$\widehat{MNP} = 50°$ and $\widehat{MPN} = 70°$. $[Nx)$ is a semi straight line holding $[NM]$.
Calculate $\widehat{PMx}$.

8 ABC is a right triangle at A such that $\widehat{BCA} = 50°$.

1°) Calculate $\widehat{CBA}$.

2°) $[AH]$ is the height relative to $[BC]$.
Calculate $\widehat{BAH}$ and $\widehat{CAH}$.

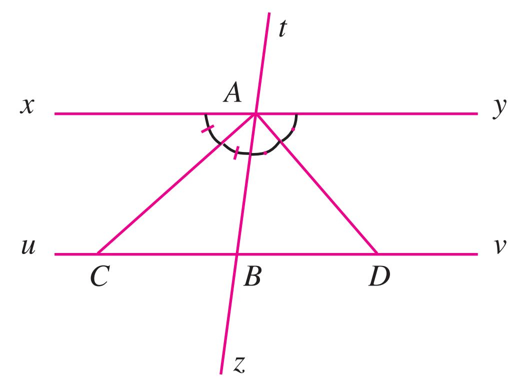
9 In the figure below, (xy) and (uv) are two parallel straight lines and (zt) is a transversal cutting them at A and B respectively.

The bisector of $\widehat{xAz}$ cuts (uv) at C.

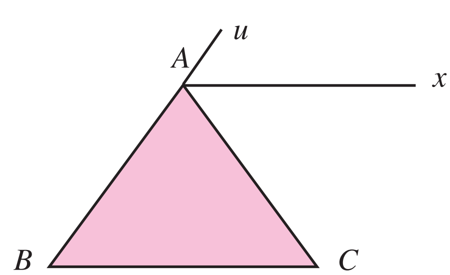
The bisector of $\widehat{yAz}$ cuts (uv) at D.

1°) Prove that triangle CAD is right.

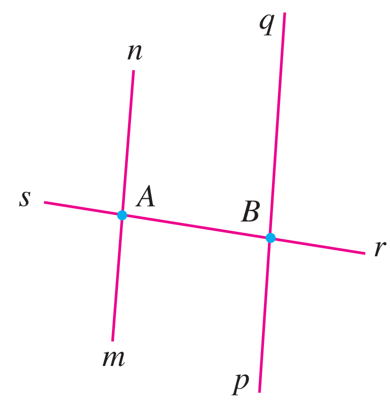
2°) Prove that triangles ABC and ABD are isosceles.



10 In the figure below, ABC is an isosceles triangle of vertex A , and $[Ax) \parallel (BC)$.
Prove that $[Ax)$ is the bisector of angle $\widehat{uAC}$.



11 Observe the adjacent figure and answer by true or false.



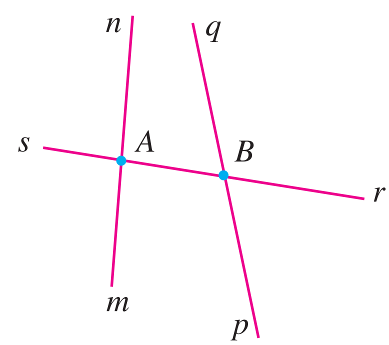
1°) $\widehat{mAB}$ and $\widehat{qBA}$ are two alternate interior angles.

2°) If (mn) is parallel to (pq), then :
$\widehat{sAn} = \widehat{ABq}$ (corresponding angles).

3°) If $\widehat{sAm} = \widehat{ABp}$, then $(mn) /\!/ (pq)$.

4°) If $\widehat{nAB} = \widehat{ABp}$, then $(mn) /\!/ (pq)$.

12 Observe the adjacent figure and Answer by true or false.



1°) $\widehat{mAB}$ and $\widehat{qBA}$ are two alternate interior angles.

2°) $\widehat{sAn}$ and $\widehat{ABq}$ are two corresponding angles.

3°) $\widehat{nAB} = \widehat{ABp}$ (alternate interior angles).

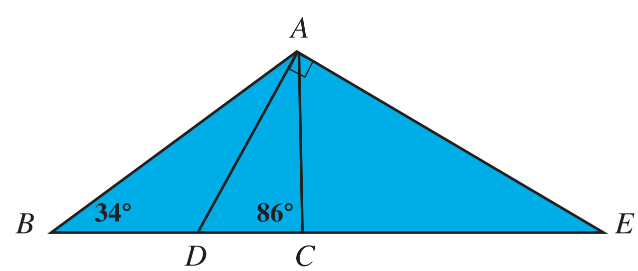
4°) $\widehat{qBr} = \widehat{nAB}$ (corresponding angles).

13 In the adjacent figure,

$[AD)$ is the bisector of $\widehat{BAC}$

and $\widehat{DAE} = 90°$.

Calculate $\widehat{AEC}$.



14 ABC is an equilateral triangle. M is any point of $[BC]$. The perpendicular at M to (BC) cuts (AB) at N and (AC) at F.
Calculate the angles of triangles BMN and ANF.

For seeking

15 ABC is an isosceles triangle of vertex A and M is any point of $[BC]$. The perpendicular bisector of $[BM]$ cuts $[AB]$ at D and that of $[MC]$ cuts $[AC]$ at E.

1°) Prove that triangle DMB is isosceles and that (MD) is parallel to (AC).

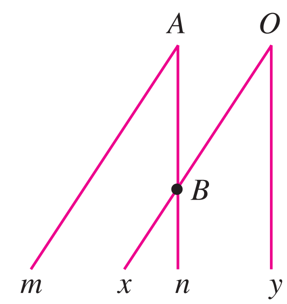
2°) Similarly, prove that (ME) is parallel to (AB).

16 $\widehat{xOy}$ and $\widehat{mAn}$ are two angles such that :

[Ox) and [Oy) are parallel to [Am) and [An) respectively.

[Ox) and [An) intersect at B.

Prove that $\widehat{mAB} = \widehat{BOy}$.



17 ABC is an isosceles triangle of vertex A and M is any point of [BC]. The perpendicular at M to (BC) cuts (AB) at I and (AC) at J.
Prove that AIJ is an isosceles triangle.

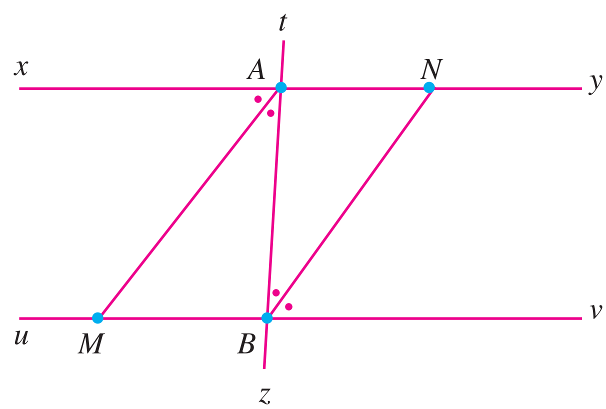
18 ABC is a right triangle at A. Let [Ax) be an interior semi-straight line of triangle ABC such that $\widehat{BAx} = \widehat{ABC}$. [Ax) cuts [BC] at M.
1°) Show that triangles AMB and AMC are isosceles.
2°) Deduce that M is the midpoint of [BC] and that $BC = 2AM$.

19 (xy) and (uv) are two parallel straight lines cut by a transversal (zt) at A and B respectively.

The bisector of angle $\widehat{xAB}$ cuts (uv) at M and the bisector of angle $\widehat{ABv}$ cuts (xy) at N.

1°) Prove that (AM) is parallel to (BN).

2°) Prove that $AM = BN$.



20 Let ABC be any triangle. The bisector of $\widehat{BAC}$ cuts [BC] at D. M is a point of [AC]. The parallel to (AD) passing through M cuts (AB) at P.
Prove that triangle AMP is isosceles.

21 (D) and (D') are two fixed parallel lines; M and N are two variable points on (D) and (D') respectively. The perpendicular at O, the midpoint of [MN], cuts (D) and (D') at E and F respectively.
Prove that $OE = OF$.

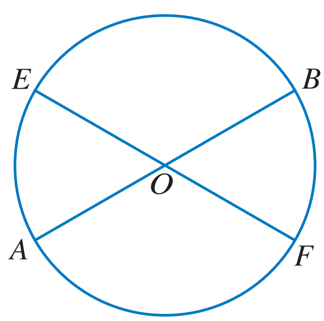
22 SAC is any triangle. The bisector of $\widehat{ASC}$ cuts $[AC]$ at O. The parallel drawn at O to (SA) cuts $[SC]$ at N.

1°) Prove that triangle SON is isosceles.

2°) The parallel drawn at N to (SO) cuts (AC) at I.

Prove that $[NI)$ is the bisector of angle $\widehat{ONC}$.

23 $[AB]$ and $[EF]$ are two diameters of the circle $C\,(O\,;R)$.



1°) Compare the triangles OAE and OFB .

2°) Deduce that (EA) is parallel to (FB) .

3°) Prove that (AF) is parallel to (EB) .

24 $MNPQ$ is a rectangle and I is the midpoint of $[MN]$.

1°) Prove that the two triangles QIM and PIN are congruent .

2°) Let $[Iu)$ be the bisector of $\widehat{PIQ}$.

Prove that $[Iu)$ is perpendicular to $[MN]$.

3°) Prove that $[Iu)$ is parallel to (MQ) .

Deduce that $\widehat{MQI} = \frac{1}{2}\,\widehat{QIP}$.

25 **1°)** Construct quadrilateral $SAMI$ such that (SA) and (SI) are parallel to (IM) and (AM) respectively.

2°) Prove that the two triangles SAI and MAI are congruent .

Deduce that $SA = IM$.

3°) The bisector of $\widehat{ASI}$ cuts (AM) at E .

a) Prove that SAE is isosceles .

b) Deduce that $AE = IM$.

26 ABC is any triangle. $[BM]$ and $[CN]$ are the medians relative to $[AC]$ and $[AB]$ respectively. E is the symmetric of B with respect to M and F is the symmetric of C with respect to N.

1°) Prove that (AE) is parallel to (BC).

2°) Prove that (AF) is parallel to (BC).

3°) Deduce that E, A and F are collinear.

27 ABC is an isosceles triangle of vertex A. $[BM]$ and $[CN]$ are the medians relative to $[AC]$ and $[AB]$ respectively.

1°) Show that AMN is isosceles.

Deduce that (MN) is parallel to (BC).

2°) Let I be the symmetric of N with respect to M.

Show that (CI) is parallel to (AN) and that $CI = AN$.

3°) Show that $NI = BC$ and deduce that $BC = 2MN$.

28 $LMNO$ is a quadrilateral such that $LM = ON = 5$ cm , $MN = OL = 3$ cm .
(LM) is parallel to (ON) and (OL) is parallel to (MN) . The parallel to (MO) at L cuts (ON) at B and (MN) at A .

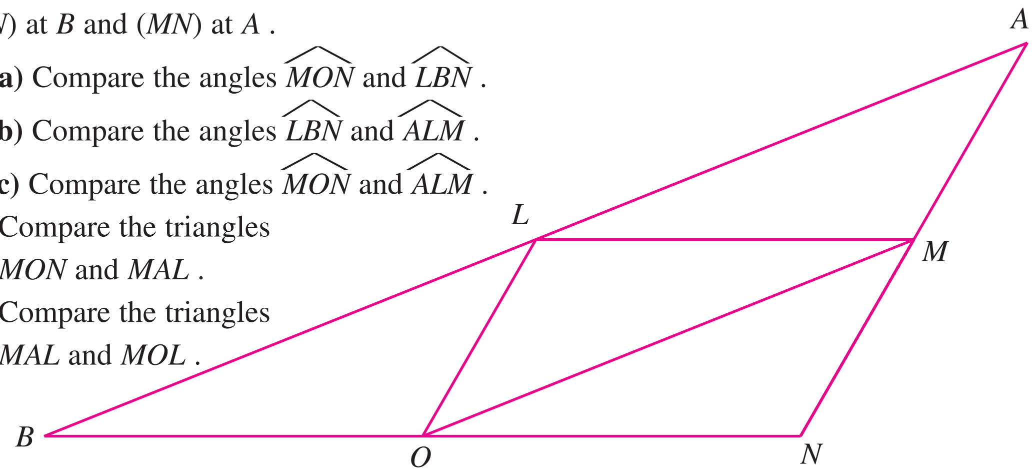
1°) **a)** Compare the angles $\widehat{MON}$ and $\widehat{LBN}$.

b) Compare the angles $\widehat{LBN}$ and $\widehat{ALM}$.

c) Compare the angles $\widehat{MON}$ and $\widehat{ALM}$.

2°) Compare the triangles MON and MAL .

3°) Compare the triangles MAL and MOL .



29 1°) Construct triangle ABC such that $BC = 4$ cm , $\widehat{ACB} = 40°$ and $\widehat{ABC} = 60°$.

2°) **a)** Construct the semi-straight line $[Ax)$ exterior to triangle ABC, such that $\widehat{CAx}=40°$.

b) Justify why $[Ax)$ is parallel to (BC) .

3°) Let D be a point of $[Ax)$ such that $AD = BC$.
Show that the two triangles ABC and ADC are congruent .

4°) **a)** Let F be a point outside of the triangle, such that $\widehat{BAF} = 60°$.

b) Is (AF) parallel to (BC) ? Why ?

c) Prove that points A , F and D are collinear .

30 On the perpendicular bisector (xy) of segment $[AB]$ and on opposite sides of this segment, take two points M and N. Consider on $[MA]$ and $[MB]$, respectively, the points C and D such that $MC = MD$. Consider on $[NA]$ and $[NB]$, respectively, the points E and F such that $NE = NF$.

1°) Show that $CA = DB$ and that (CD) is parallel to (AB).

2°) Show that $EA = FB$ and that (EF) is parallel to (AB).

3°) Deduce that (EF) is parallel to (CD).

4°) Show that the two triangles ACE and BDF are congruent.

31 ABC is a right triangle at B . Let D be on (BC) such that B is the midpoint of $[CD]$.

1°) **a)** What does (AB) represent to $[CD]$? Justify your answer.

b) Deduce the nature of triangle ACD .

2°) Draw the parallel to (BC) passing through A .
Take point M on this parallel such that $AM = BC$, M and C being on the same side of (AB) .

a) Show that $\widehat{MAC} = \widehat{ACB}$.

b) Show that the two triangles ABD and AMC are congruent .

32 ABC is an isosceles triangle of vertex A , M is the midpoint of $[BC]$ and E is the symmetric of M with respect to B .

The perpendicular at E to (BC) cuts (AB) at F .

1º) What does (AM) represent in triangle ABC ?

2º) Show that the two triangles ABM and EBF are congruent .

List their corresponding equal parts.

3º) Show that (MF) is parallel to (AE).

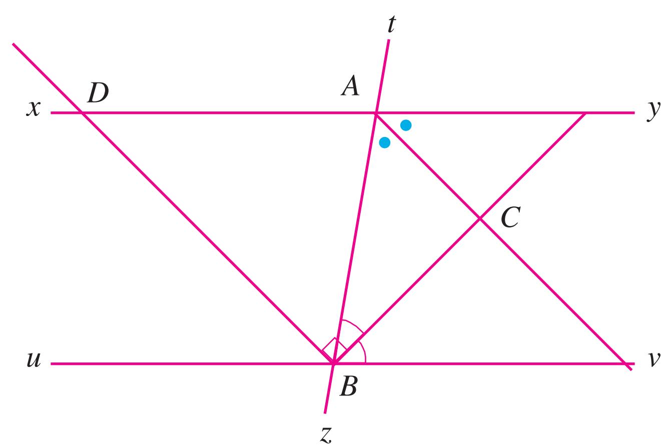
33 (xy) and (uv) are two parallel straight lines cut by a transversal (zt) at A and B respectively.

The bisector of $\widehat{yAB}$ and that of $\widehat{vBA}$ intersect at C.

1º) Show that triangle ACB is right.

2º) The perpendicular drawn through B to (BC) cuts (xy) at D.

Show that $[BD)$ is the bisector of $\widehat{ABu}$.



34 ABC is any triangle such that $AB = 9$ cm ; $AC = 8$ cm and $BC = 6.5$ cm .

$[Ax)$ is the bisector of $\widehat{BAC}$.

(d) is the perpendicular drawn at A to $[Ax)$.

The parallel at C to (AB) cuts (d) at E .

F is the symmetric of E with respect to A .

1º) Show that $\widehat{FAB} = \widehat{EAC}$.

2º) Show that triangle AEC is isosceles .

3º) The parallel to (d) at C cuts (AB) at G and $[Ax)$ at H .
Show that triangle ACG is isosceles .

4º) Show that triangles AEC and AGC are congruent .

5º) Deduce that $CG = AF$.

6º) Show that triangles AFG and ACG are congruent .

7º) Show that (FG) is parallel to (AC) .

8º) M is the orthogonal projection of F on (CG) .
N is the orthogonal projection of E on (CG) .
Show that $GM = CN$.

9º) Deduce that H is the midpoint of $[MN]$.

TEST

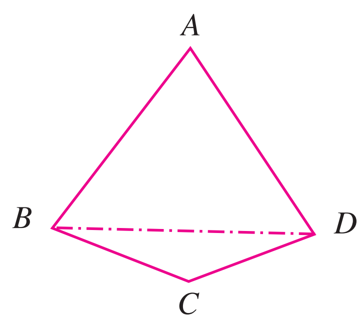
1 $\widehat{xAy}$ is a given angle, AM is the bisector of $\widehat{xAy}$.

The perpendicular bisector of $[AM]$ cuts $[Ax)$ at B.

1º) Show that (MB) is parallel to (Ay).

2º) Calculate $\widehat{ABM}$ when $\widehat{xAy} = 60°$. **(2 points)**

2 In the figure below,



Show that :

$\widehat{DAB} + \widehat{ABC} + \widehat{BCD} + \widehat{CDA} = 360°$. **(2 points)**

3 $[AB]$ and $[CD]$ are two equal parallel segments. $[AD]$ and $[BC]$ intersect at O.

Show that O is the midpoint of $[BC]$ and $[AD]$. **(2 points)**

4 MEN is any triangle. The bisectors of angles $\widehat{MEN}$ and $\widehat{MNE}$ intersect at I. The parallel drawn from I to (EN) cuts (ME) at A and (MN) at B.

1º) Show that triangles AIE and BIN are isosceles. **(2points)**

2º) Show that the perimeter of triangle MAB is equal to $ME + MN$. **(2 points)**

21 PROPORTIONS

Objectives

- Recognizing a proportionality situation.
- Recognizing a proportion.
- Transforming a proportion to obtain another.
- Calculate the fourth proportional.
- Using the calculation of the fourth proportional in problems.

CHAPTER PLAN

1 DIRECTLY PROPORTIONAL MAGNITUDES

Activity

At the butcher, it is sufficient to type the price of a kilogram for which the price of a weighed piece is typed immediately.

1º) Complete the following table of proportionality.

Mass in kg	1	0.1	0.2	0.3	0.4	2		3
Price in L.L.	325						162.5	

x…

2º) How can you calculate the terms of the second row from the ones of the first row ?

3º) How can you calculate the terms of the first row from those of the second row ?

4º) Are the prices proportional to the masses ?

Definition

Two **magnitudes** are **directly proportional** when we obtain the value of the second magnitude by multiplying with the same number those of the first; in other words, each corresponding pair of numbers give the same quotient (**proportionality cœfficient**).

EXAMPLE

The following table :

Number of pens	**3**	**5**	**10**	**13**	**15**
Price in L.L.	**18**	**30**	**60**	**78**	**90**

x…

proves that the price of pens is proportional to their number.

$\frac{18}{3} = \frac{30}{5} = \frac{60}{10} = \frac{78}{13} = \frac{90}{15} = 6.$

6 is the proportionality cœfficient.

However, the table below :

Age in years	10	12	15	18	20
Height in cm	**140**	**150**	**160**	**170**	**180**

proves that the height is not proportional to the age.

$\frac{140}{10} \neq \frac{150}{12}$.

Remark :

« *a* , *b* and *c* are **respectively proportional** (or **directly proportional**) to 5 ; 2 and 3 » , means :

$$\frac{a}{5} = \frac{b}{2} = \frac{c}{3}$$

Application 1

1º) Is the following table a proportionality table ?

Mass in kg	1	0.1	0.2	0.3	0.5	2	2.5	3
Price in L.L.	65	6.5	13	19.5	32.5	130	162.5	195

2º) How do you calculate the numbers of the second row from those of the first row ?

3º) How do you calculate the numbers of the first row from those of the second row ?

4º) What is the proportionality cœfficient ?

5º) Complete : The prices are … to the masses.

2 PROPORTION

Activity

$a = 2.4$, $b = 3$, $c = 2.72$ and $d = 3.4$ are 4 decimals.

a) The ratio of a to b is : $\frac{a}{b} = \frac{2.4}{3} = 0.8$;

calculate the ratio of b to a, c to d and d to c.

b) Compare $\frac{a}{b}$ and $\frac{c}{d}$ then complete : $\frac{a}{b} \ldots \frac{c}{d}$.

c) Compare $a \times d$ and $b \times c$.

d) Compare $\frac{b}{a}$ and $\frac{d}{c}$ then complete : $\frac{b}{a} \ldots \frac{d}{c}$.

e) Complete :

$$\frac{2.4}{3} = \frac{2.72}{\ldots} \quad ; \quad \frac{\ldots}{3} = \frac{2.72}{3.4} \quad ; \quad \frac{2.4}{\ldots} = \frac{2.72}{3.4} \quad ; \quad \frac{2.4}{3} = \frac{\ldots}{3.4}.$$

f) Calculate the ratio of a to c and that of b to d then complete : $\frac{a}{c} \ldots \frac{b}{d}$.

Definition

A proportion is an equality of two **ratios** $\frac{a}{b}$ and $\frac{c}{d}$, denoted by : $\boldsymbol{\frac{a}{b} = \frac{c}{d}}$.

$$\begin{array}{rcccl} \text{1}^{\text{st}}\text{ term} & \longleftarrow & a & & c & \longrightarrow & \text{3}^{\text{rd}}\text{ term} \\ & & \overline{b} & = & \overline{d} & & \\ \text{2}^{\text{nd}}\text{ term} & \longleftarrow & & & & \longrightarrow & \text{4}^{\text{th}}\text{ term} \end{array}$$

The **first** and the **fourth** terms are the **extremes**.

The **second** and the **third** terms are the **means**.

EXAMPLE

$\frac{3}{4}$ and $\frac{6}{8}$ are two equal ratios ; they form a proportion : $\frac{3}{4} = \frac{6}{8}$.

3 and 8 are respectively the first and the fourth terms: they are the extreme terms.

4 and 6 are respectively the second and the third terms: they are the mean terms.

Application 2

$\frac{2.5}{25} = \frac{20}{200}$ is a proportion.

1°) List its extremes terms and its means.

2°) Complete :

… is the third term of this proportion.

3 PROPERTIES OF A PROPORTION

1°) In a proportion, **the product of the means is equal to the product of the extremes.**

In $\frac{a}{b} = \frac{c}{d}$, $a \times d = b \times c$.

EXAMPLE

In $\frac{3}{4} = \frac{6}{8}$, $3 \times 8 = 4 \times 6$.

2°) Consider the proportion $\frac{a}{b} = \frac{c}{d}$,

• if we **permute the means**, we obtain a new proportion $\frac{a}{c} = \frac{b}{d}$.

EXAMPLE

If we permute the means of the proportion $\frac{3}{4} = \frac{6}{8}$,

we obtain the proportion $\frac{3}{6} = \frac{4}{8}$.

• if we **permute the extremes**, we obtain a new proportion $\frac{d}{b} = \frac{c}{a}$.

EXAMPLE

If we permute the extremes of the proportion $\frac{3}{4} = \frac{6}{8}$,

we obtain the proportion $\frac{8}{4} = \frac{6}{3}$.

• if we **inverse the ratios**, we obtain a new proportion $\frac{b}{a} = \frac{d}{c}$.

EXAMPLE

If we inverse the ratios of the proportion $\frac{3}{4} = \frac{6}{8}$,

we obtain the proportion $\frac{4}{3} = \frac{8}{6}$.

Application 3

1 - a) Complete so as to have a proportion : $\frac{2}{6} = \frac{\dots}{24}$.

b) List the means of the preceding proportion. What proportion do you obtain if you permute them ?

c) How do you obtain the proportion $\frac{24}{6} = \frac{8}{2}$ from the one obtained in **a)** ?

d) Invert the ratios of the proportion obtained in **a)** ; do you obtain a proportion?

2 - a) By multiplying the means and the extremes, prove that $\frac{2.5}{0.7} = \frac{32.5}{9.1}$ is a proportion.

b) Without any calculation, write all the proportions that you can obtain from the above proportion.

4 FOURTH PROPORTIONAL

Observe the way to calculate, in each of the proportionality tables below, the missing number x.

1 -

51	x
12	5

From this table we get the proportion :

$$\frac{12}{51} = \frac{5}{x},$$

the product of the extremes is equal to the product of the means, thus :

$$12 \times x = 5 \times 51$$
$$12\,x = 255$$
$$x = \frac{255}{12}$$

therefore $x = 21.25.$

2 -

1.5	3
x	9

From this table we get the proportion :

$$\frac{x}{1.5} = \frac{9}{3};$$

the product of the extremes is equal to the product of the means, thus :

$$3 \times x = 9 \times 1.5$$
$$3\,x = 13.5$$
$$x = \frac{13.5}{3}$$

therefore $x = 4.5.$

3 -

2.5	3.5
23.5	x

From this table we get the proportion :

$$\frac{23.5}{2.5} = \frac{x}{3.5},$$

the product of the means is equal to the product of the extremes, thus :

$$2.5 \times x = 23.5 \times 3.5$$
$$2.5\,x = 82.25$$
$$x = \frac{82.25}{2.5}$$

therefore $x = 32.9.$

4 -

x	50
1.6	3.2

From this table we get the proportion :

$$\frac{1.6}{x} = \frac{3.2}{50} ,$$

the product of the means is equal to the product of the extremes, thus :

$$3.2 \times x = 1.6 \times 50$$

$$3.2\,x = 80$$

$$x = \frac{80}{3.2}$$

therefore $x = 25.$

In each of the preceding proportionality, the missing number is called the fourth proportional.

5 - To determine the **fourth proportional** of three given numbers 2 , 3 and 4, for example, is to calculate the value of the fourth term of this proportion :

$$\frac{2}{3} = \frac{4}{\boldsymbol{x}}$$

$$2 \times x = 3 \times 4$$

$$2x = 12$$

$$x = \frac{12}{2}$$

therefore $x = 6.$

Application 4

1 - Calculate x in each of the following proportions.

$\frac{x}{2} = \frac{3}{5}$; $\frac{3}{x} = \frac{4}{3}$; $\frac{2}{15} = \frac{x}{4}$; $\frac{1}{6} = \frac{3}{x}$; $\frac{1.5}{9} = \frac{4.5}{x}$.

2 - Find the fourth proportional of the three numbers :

3.2 ; 50 and 9.6.

5 THE TRIPLE RULE

2 erasers cost 800L.L. ; what is the price of 5 erasers ?

1st method

To complete the proportionality table below (you start by finding the proportionality cœfficient).

Number of erasers	2	5		
price in L.L.	800	x		

...

The proportionality cœfficient is $800 : 2 = 400$.

The price of 5 erasers is $5 \times 400 = 2000$

2000 L.L.

From the preceding table, the fourth proportional x of the three numbers : 2 , 800 and 5 is given by :

$$\frac{2}{800} = \frac{5}{x}$$

$$2 \times x = 5 \times 800$$

$$2x = 4000$$

$$x = \frac{4000}{2} \;;$$

therefore $x = 2000$.

The price of 5 erasers is 2000 L.L.

2nd method

<u>(Triple rule)</u>

To 2 we correspond 800 or 2 ⟶ 800

To 5 we correspond x or 5 ⟶ x

We say that we express the given problem by the **triple rule**.

2 and 5 are directly proportional to 800 and x.

$$\frac{2}{800} = \frac{5}{x}$$

$$2 \times x = 5 \times 800$$

$$2x = 4000$$

$$x = \frac{4000}{2} ;$$

therefore $x = 2000$.

The price or 5 erasers is 2000 L.L.

Application 5

Solve by the triple rule :

a) To 2 corresponds 5 ; to 12 corresponds what ?

b) 10 litres of fuel cost 6200 L.L. What is the price of 20 litres ?

c) The ribbon of a tape of 60 minutes measures 284 m in length. What is the length of a tape of 150 minutes?

Exercises and problems

For testing the knowledge

1 Which of the following tables is a proportionality table ?

1	11	22
3	33	66

2	6	8	10
5	15	25	30

3	30	9	12	15
1.3	13	3.9	5.2	6.5

2 A car moves uniformly at the same speed. The duration of the journey is proportional to the distance traveled. Copy and complete the following table.

Distance (in km)	150	75	225	37.5		
Time (in min)	120				80	40

3 Calculate x and y where 6, x and 5 are directly proportional to 4 ; 8 and y.

4 The magnitude x is directly proportional to the magnitude y .
Knowing that $x = 8$ when $y = 6$, calculate x when $y = 10$.

5 Write all the proportions whose terms are the factors of the following two equal products : $12 \times 25 = 6 \times 50$

6 Can you form a proportion with the numbers : 4 ; 17 ; 12 and 51?

7 Calculate the ratio of the length L to the width l of a rectangular field knowing that $L=$ 0.350 km and $l=70$m.

8 To manufacture 30 kg of flour, we need 40 kg of corn.
How much corn should we have to manufacture 1 kg of flour ?

9 In each of the following problems, two magnitudes are included.
a) What are these two magnitudes ?
b) Are these magnitudes proportional ?
c) Solve the problem if possible.

1 - We need 250 g of spaghettis for 4 persons.
How many g are needed for 10 persons?

2 - At the age of 13, Malek measures 1.40 m. How tall will he be at 30 years ?

3 - A square of side of 5 cm has an area of 25 cm^2.
What is the area of a square of side 15 cm ?

10 Calculate in each case, the fourth proportional.

1°)

14	9
2.8	x

2°) $\dfrac{7.5}{x} = \dfrac{135}{9}$

3°) $\dfrac{0.5}{0.16} = \dfrac{x}{4}$

4°) $\dfrac{9}{3.6} = \dfrac{a}{5}$

5°) $\dfrac{x}{5.4} = \dfrac{12}{8.1}$

6°) $\dfrac{t}{36} = \dfrac{13.5}{4}$

11 Calculate in each case, the fourth proportional of the three given numbers.

1°) 5 ; 8 and 16

2°) 1.5 ; 4 and 0.8

3°) $\dfrac{1}{3}$; $\dfrac{1}{4}$ and $\dfrac{1}{5}$

12 80 liters of fuel cost 48 000 L.L. How much do 90 liters cost ?

13 A heating installation consumes regularly 12 liters of oil in 24 hours.
How much does it consume in 250 hours ?

14 By moving at a constant speed, Jad traveled on a bicycle 100 km in 6 hours.
What distance did he travel in 3 hours ?

15 **a)** What is the perimeter of a square whose side measures 5 cm ? 14 cm ? 17 cm ?

b) What is the measure of a side of a square when its perimeter measures 48 cm ? 44 cm ?

When we make a map (or a figure), «a scale» is used; there is a proportionality between the true distances and those of the map.

Determining a scale is to calculate a proportionality

cœfficient = $\dfrac{\textbf{distance on the map}}{\textbf{true distance}}$

distance on the map = proportionality cœfficient × true distance.

16 On a plane of scale $\dfrac{1}{200}$:

a) Which true distance corresponds to a segment of 5.2 cm?

b) What is the length of a segment which represents a true distance of 4.8 cm?

17 On a plane of scale 25 :

a) which true distance corresponds to a segment of 7.5 cm ?

b) what is the length of a segment which represents a true distance of 0.8 mm ?

18 The length of a road is 600 m. Calculate its length on a map of scale $\frac{1}{80\,000}$.

19 What scale should we choose to represent a square of side 80 cm by a square of side 5 cm ?

- **Calculate a percentage is to calculate the fourth proportional.**
- **To get the a% of a number, we multiply this number by $\frac{a}{100}$.**

20 Of 450 students in a school, 126 are in the intermediate cycle.
What is the percentage of students in this cycle ?

21 Of the 160 pages of a magazine, 40 are occupied by publicity. What is the percentage of the total number of pages representing this part ?

22 Answer by true or false.

1º) The equality $\frac{4}{3} = \frac{8}{6}$ is a proportion.

2º) The proportion $\frac{4}{3} = \frac{6}{4.5}$ is obtained by permuting the means of the proportion $\frac{4}{6} = \frac{3}{4.5}$.

3º) If $\frac{a}{b} = \frac{c}{d}$, then $a \times b = c \times d$.

4º) If the numbers f, a and b are directly proportional to 4 , 3 and 2 then $\frac{f}{4} = \frac{a}{3} = \frac{b}{2}$.

5º) If $a \times b = c \times d$, then a , b , c and d are the terms of a proportion.

6º) If we triple the sides of a triangle, then the perimeter is tripled.

7º) The perimeter of a square is directly proportional to the side of this square.

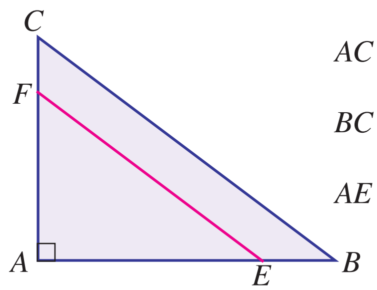
8º) If we double the side of a square, then the area is doubled.

9º) The numbers 4 ; 240 ; 15 and 20 form a proportion.

10º) Taking 25% of a number is to multiply it by $\frac{1}{4}$.

For seeking

23



$AB = 4$ cm

$AC = 3$ cm

$BC = 5$ cm

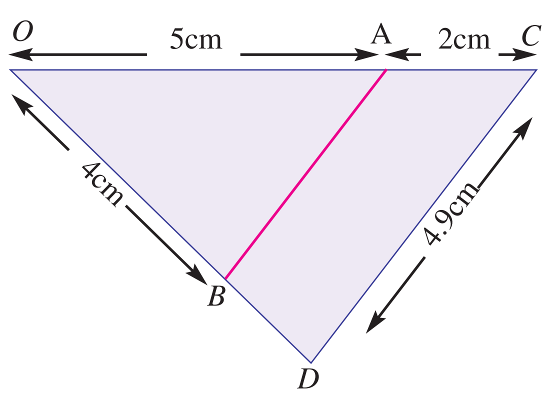
$AE = 3$ cm

Knowing that the lengths AB, AC and BC are directly proportional to the lengths AE, AF and EF, calculate AF and EF.

24 Each bag of candies contain 6 Easter eggs and 11 chocolate fish. There are 186 Easter eggs.

How many chocolate fish are there ?

25 In the figure below, the lengths OA , OB and AB are directly proportional to the lengths OC , OD and CD . Calculate OD and AB.



26 Copy and complete the proportionality table below :

Number of CD's	3	11
Price (in L.L.)	210	

This table is done to solve a problem. What could the statement be ?

27 Do the plane of a rectangular dining room of 5.40 m and 4.50 m to the scale $\frac{1}{75}$.

At a constant speed v, the distance traveled d is proportional to the time t of the journey.

$$d = v \times t \qquad v = \frac{d}{t} \qquad t = \frac{d}{v}.$$

28 Majed :«I have just covered 120 km in 1 h 15 min ».
Walid : «I needed 1 h 45 min to cover 175 km».
Hadi : «I did 133 km in 1 h 24 min ».
Mazen : «I needed 40 min to cover 64 km ».

Who is the fastest driver ?

29 The surface of the earth is $511\,966\,000 \text{ km}^2$. The oceans have a surface of $362\,030\,000 \text{ km}^2$.
What percentage does the surface of the earth represent to the oceans' surface ?

30 A person has a monthly budget of 900 795 L.L. He spends 12% for a rent.

a) What is the amount of the rent ?

b) This month, this person spent 135 119.25L.L. on leisure.
What percentage of monthly budget does this sum represent ?

31 The output of a tap is 150 liters in 12 minutes.

a) How much time is needed to fill a tube of 600 liters ?

b) Can a tank of 1800 liters be filled in 2 h 30 min ?

32 An airplane leaves London at 10 h 15 min and lands in Rome at 12 h 27 min.

a) What is the duration of the flight ?

b) On a Europe map, of scale $\frac{1}{35\,000\,000}$, the distance separating Rome from London is 41 mm.

What is the original distance from London to Rome ?

c) Calculate the average speed of the plane (assuming that it followed this line at a constant speed).

33 Two cyclists started from the same place at 8:30 a.m.
The average speed of the first is 35 km/h, and the second is equal to $\frac{4}{5}$ of that of the first.

a) Calculate the average speed of the second.
b) The first arrives to his destination at 11h 06 min.
At what distance from this point is the second found ?
c) Calculate the distance traveled by the first.
d) At what time does the second arrive ?

34 My mother sent two carpets to be cleaned. One is rectangular; with dimensions 1.8 m and 2.4 m . The other is circular, with diameter 2.1 m.

a) Calculate the area of each carpet (round the area of the circular carpet to dm^2).

b) The price of cleaning is proportional to the area of the carpet. Mother paid 145000 L.L.

How much should she pay for the other carpet ?

35 The population of a city was 25 283 inhabitants on 1 - 1 - 96 and 25 445 inhabitants on 1 - 1 - 97.

What is the percentage of the increase of this population ?

TEST

1 Calculate the fourth proportional of the numbers 9.1 ; 6.5 and 2.8. **(2 points)**

2 Write all the proportions that you can form with the numbers
9.1 ; 6.5 ; 2.8 and 2. **(3 points)**

3 The numbers : $a - 2$, $b + 3$ and 4 are directly proportional to the numbers : 1.5 ; 3 and 2.

Calculate a and b. **(4 points)**

4 What proportion do you obtain if you add 1 to both sides of the proportion $\frac{a}{b} = \frac{c}{d}$? **(2 points)**

Permute the means of the proportion. What proportion do you obtain ? **(1 point)**

5 Use the triple rule to solve the following.

a) The length of a sweater is proportional to the number of rows of sweaters.

If the length of 20 rows is equal to 6 cm, how many rows should we have for a length of 36 cm ? **(2 points)**

b) For 24 pancakes, we need 500g of flour. How much flour is needed for 72 pancakes ? **(2 points)**

6 **a)** The track of an airport measures 2.8 km. What is, in cm, the length of this track on a map of scale $\frac{1}{50\,000}$? **(2 points)**

b) Calculate the original distance in km between two villages when their distance on a map whose scale is $\frac{1}{200\,000}$ is equal to 3.5 cm. **(2 points)**

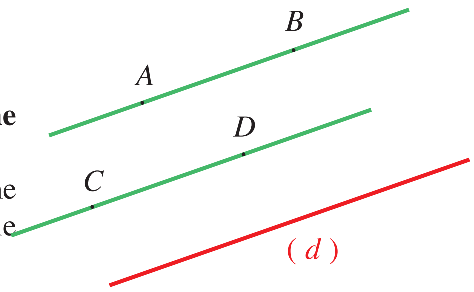
22 TRANSLATIONS

Objectives

- Defining the movement of a figure by sliding it according to a given direction.
- Defining a translation as being the sliding in a given direction, in a given sense and at a given distance.
- Tracing the image of a figure knowing the image of one of its points.
- Preserving the distances, the angles, the collinearity and the parallelism by a translation.

CHAPTER PLAN

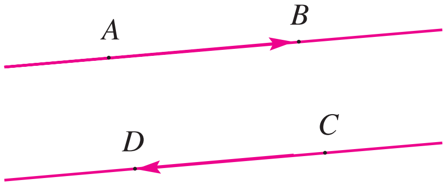
1 DIRECTION



When two straight lines are parallel, we say that they have **the same direction.**
(AB) and (CD) are parallel therefore they have the same direction. Every straight line parallel to them, for example (d), indicates this direction .

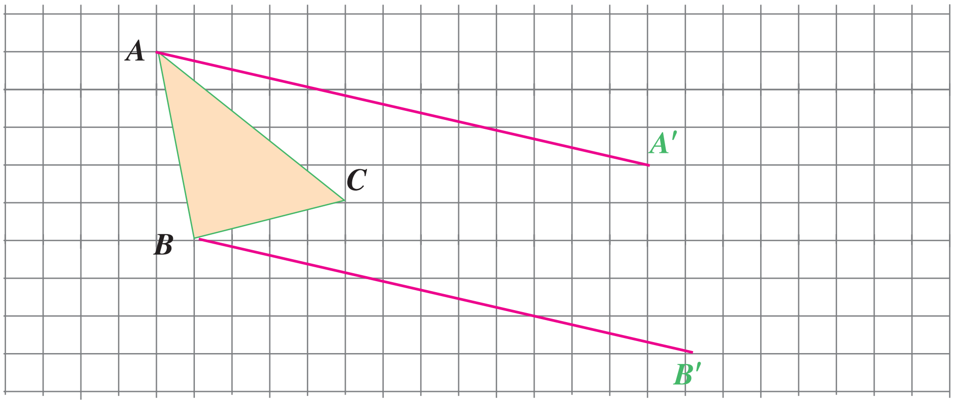
2 SENSE

A direction being indicated by the given straight line (AB), there are two **senses** to travel in this direction: either from A to B, or from B to A.
In the adjacent figure, the straight lines (AB) and (CD) have the same direction.
On (AB), the sense is that going from A to B.
On (CD), the sense is that going from C to D. These two senses are **opposite**.



3 TRANSLATION

Activity



In the figure above :
B' is the image of B obtained by sliding in the direction of (AA'), the sense going from A to A' and of length equal to AA'.

1º) Place the point C' the image of C by this sliding.

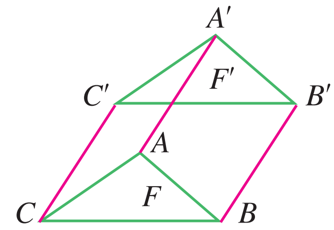
2º) Is (CC') parallel to $[BB']$? Justify .

3º) Do $[AB]$ and $[A'B']$ have the same length ? Is it the same for $[AC]$ and $[A'C']$? For $[BC]$ and $[B'C']$?

4º) Are the two triangles ABC and $A'B'C'$ congruent ?
List the equal angles of these two triangles.

Definition

The figure F' is obtained by sliding the figure F :
- in the direction of the straight line (AA'),
- in the sense of A to A',
- of length equal to AA'.



F', obtained by the sliding of F, is called the **image** of F by this **translation**, or F' is the **transformation** of F by this translation.
The points A', B', and C' are respectively the **image**s of the points A, B and C by this translation or the **transformation** of A, B and C by this translation

Properties

1º) By a translation, the image of a straight line is a straight line parallel to it.
We say that the translation preserves the parallelism.

2º) By a translation, the images of three collinear points are collinear.
We say that the translation preserves collinearity.

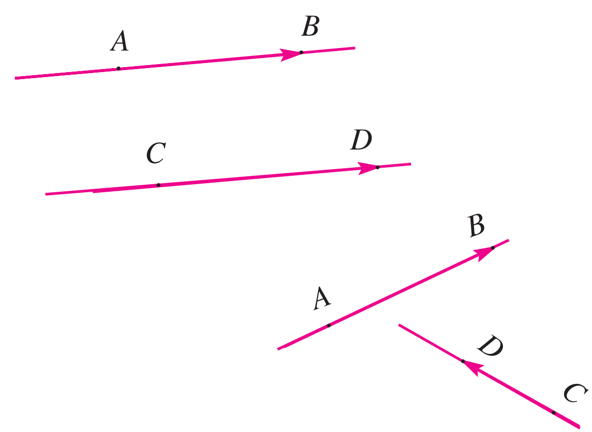
3º) By a translation, the image of a segment is a segment having the same length.
We say that the translation preserves the lengths.

4º) By a translation, the image of an angle is an angle having the same measure.
We say that the translation preserves the angles.

Exercises and problems

For testing the knowledge

1 Answer by true or false.



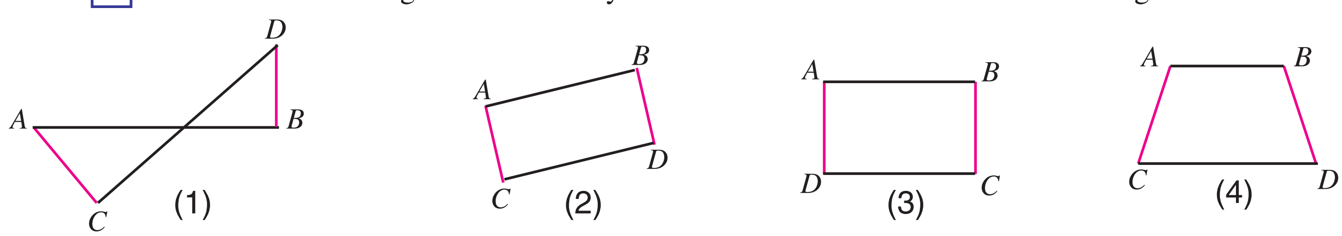
1º) a) The straight lines (AB) and (CD) have the same direction.

b) The sense from A to B is the same as that from C to D.

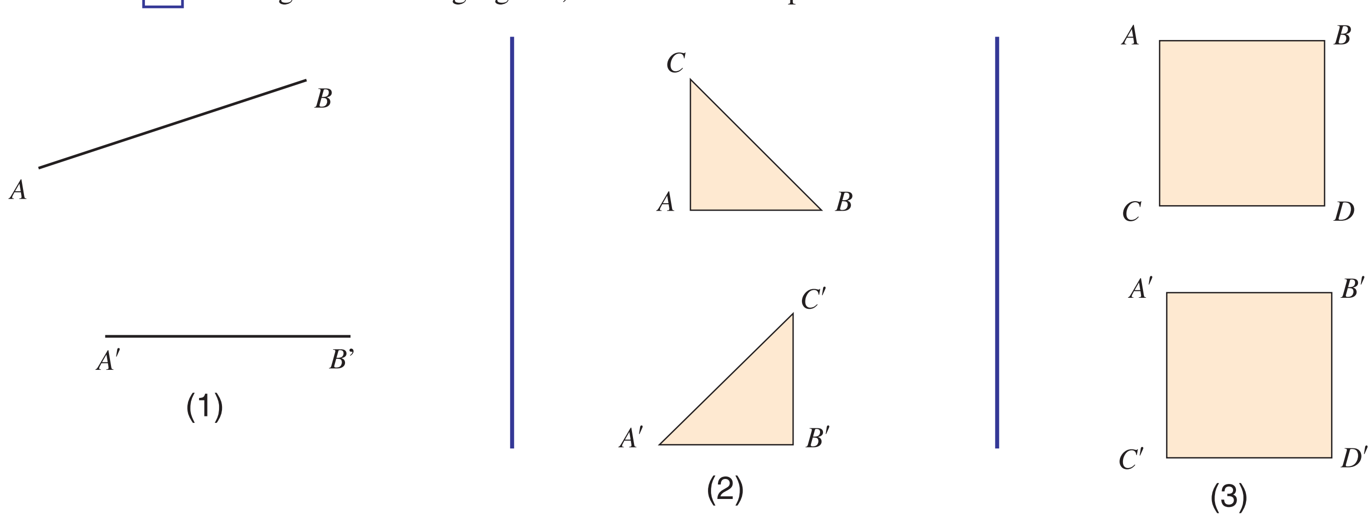
2º) a) The straight lines (AB) and (CD) have the same direction.

b) The sense from A to B is the opposite from that of C to D.

2 C and D are the images of A and B by a same translation. Indicate the correct figure.



3 Among the following figures, which one corresponds to a translation.



4 *ABCD* is a rectangle. Recopy and complete the following table.

The translation	which moves A to B	which moves B to C	which moves … to A
The point	D		C
The image		D	B

5 *ABCD* is a rectangle of dimensions 5cm and 3cm and of center O. Construct the image of *ABCD* by the following translations :
a) t_1 which moves A to B ; **b)** t_2 which moves A to C ; **c)** t_3 which moves B to O.

6 *ABC* is an equilateral triangle of side 2cm. Construct the image of *ABC* by the following translations :
a) t_1 which moves A to B **b)** t_2 which moves C to A **c)** t_3 which moves B to C.
Compare the four triangles.

7 Answer by true or false :

1º) The translation does not preserve lengths.

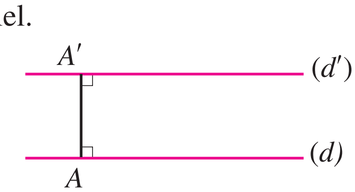
2º) The transformation of any triangle is an equilateral triangle.

3º) The image of a triangle by a translation is a triangle congruent to it.

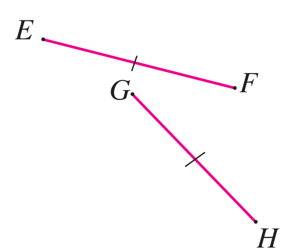
4º) The transformation of a right angle is a right angle.

5º) The images by a translation of two parallel segments are not parallel.

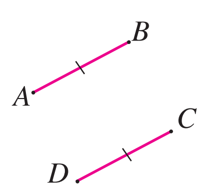
6º) A' is the image of A by a translation.
It is the only translation which transforms (d) into (d').



7º) If $EF = GH$, the translation which transforms E into G, transforms F into H.



8º) The translation which transforms A into B, transforms C into D.

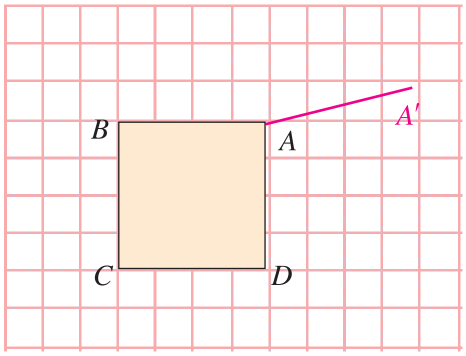


For seeking

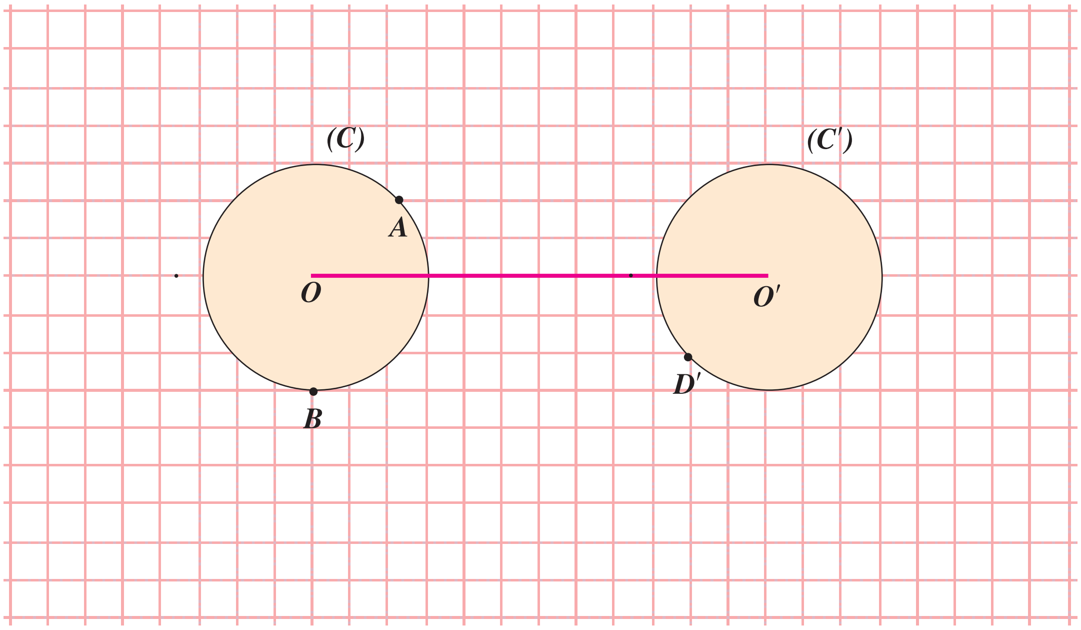
8 **1°)** Reproduce on a paper the adjacent square.

2°) A' being the image of A by a translation, construct the images B', C' and D' of B, C and D by this translation.

3°) What is the nature of quadrilateral $A'B'C'D'$?



9 (C) and (C') are two circles of respective centers O and O' having equal radii. Let t be the translation which maps O to O'.



1°) Construct the images of A and B by this translation. Where are these points found ?

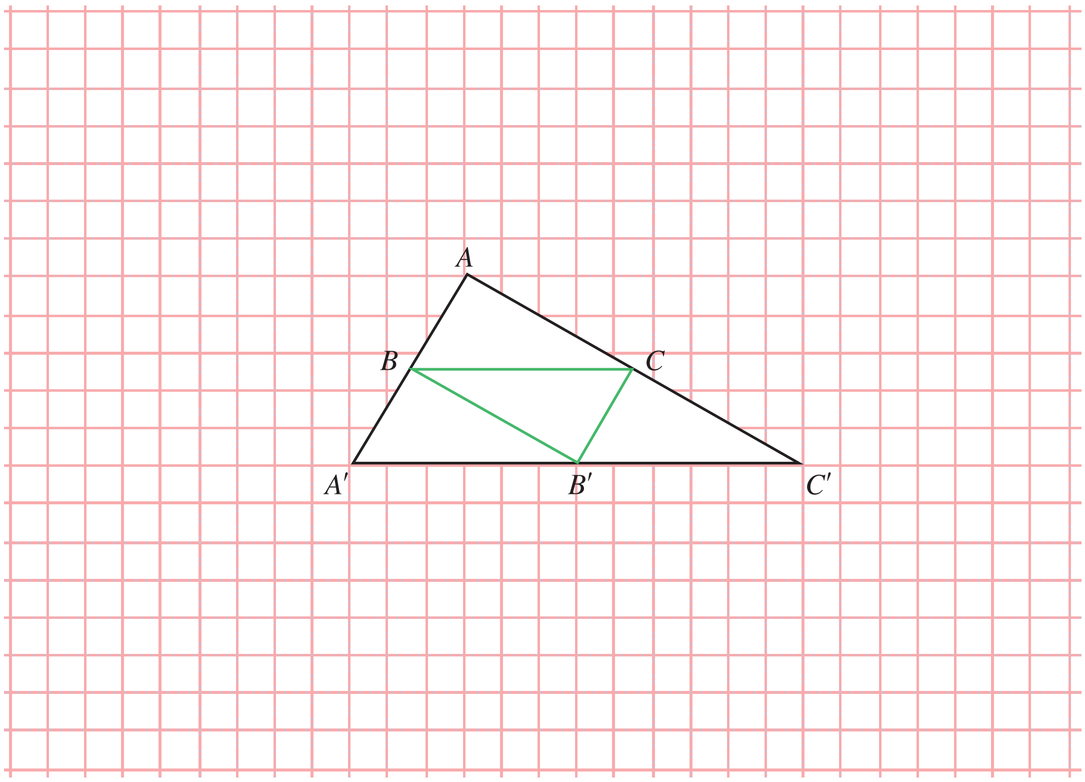
2°) D' is a point of (C'). Construct the point D where its image by the translation t is D'. Where is D found ?

3°) What is then the image of circle (C) by this translation?

4°) A parallel to (OO') cuts (C) and (C') respectively at E, F, G and H.

What are the points that correspond by the same translation?

10 Observe the figure below.



1º) We construct the image of triangle ABC by the translation which moves A to B. Which triangle do we obtain ?

2º) We also construct the image of triangle ABC by the translation which moves A to C. What triangle do we obtain ?

3º) Why do we have $(AB) \parallel (CB')$ and $AB = CB'$?

4º) Without justification, name the image of triangle $BA'B'$ by the translation which moves B to C.

11 ABC is any triangle. From a point E of $[AB]$, we draw the parallel to $[BC]$ which cuts $[AC]$ at F.

1º) Construct the image K of B by the translation which moves E to F.

2º) Determine the point J of $[BC]$ where the image by this translation is C.

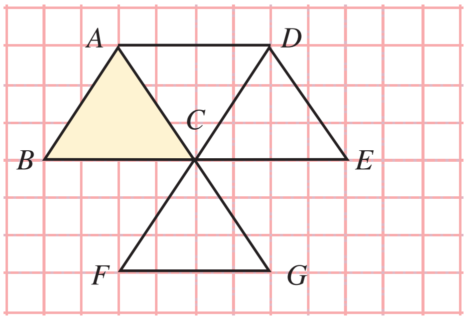
3º) Show that $EF = BK = JC$

4º) If I is the midpoint of $[BC]$, show that I is also the midpoint of $[KJ]$.

TEST

1 In the adjacent figure, the four triangles are congruent.

1º) Answer by true or false.



a) Consider the translation which moves A to B.

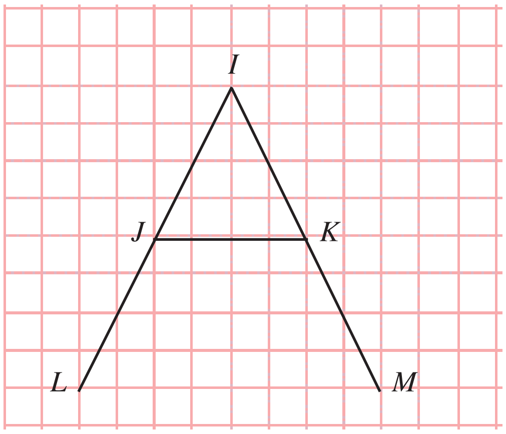
- F is the image of C. **(1 point)**
- E is the image of D. **(1 point)**
- $[FG]$ is the image of $[BC]$. **(1 point)**

b) Consider the translation from A to D.

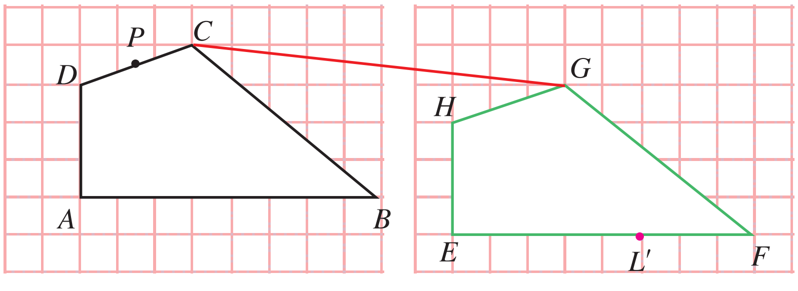
- E is the image of B. **(1 point)**
- $[FG]$ is the image of $[CE]$. **(1 point)**
- $[AG]$ is the image of $[DE]$. **(1 point)**

2º) Name the triangles obtained from triangle ABC by two translations which should be defined **(2 points)**

2 Observe the following figure representing the letter A.
Copy and construct the image of this figure by the translation which moves I to L.
(5 points)



3 $EFGH$ is the image of $ABCD$ by the translation which moves C to G.



a) Find the image P' of P by this translation. Justify. **(2 points)**

b) Let L′ be a point of $[EF]$. Construct point L where L' is its image by this translation. Justify. **(2 points)**

c) Justify why $CG = DH = AE = BF$. **(3 points)**

23 FIXED POINTS AND VARIABLE POINTS CONSTRUCTIONS

Objectives

- Differentiate between a fixed point and a variable point .
- The Locus of a point is a fixed curve (line, circle or other) on which varies a point verifying certain properties .
- Using the Locii in construction .

CHAPTER PLAN

1 DEFINITION

• A point whose position does not change is called a fixed point, or else it is called a variable point.

• A line (straight line or curve) whose position does not change is called fixed or else it is said to be variable.

• A segment which has a given length is called a segment of constant length.

2 REMARKABLE LINES

1°) Straight lines parallel to a given fixed straight line

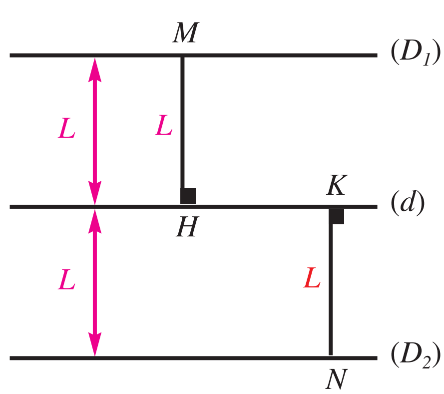
Activity

Let (d) be a fixed straight line. A and E are two points on opposite sides of (d) and at a distance of 2 cm from (d), i.e the segments $[AH]$ and $[EK]$ perpendicular to (d) have the same length 2 cm.

1°) Place two points B and C on the side of A with respect to (d) and at 2 cm from (d). Are the points A, B and C collinear ? If yes, how is this straight line with respect to (d)?

2°) **a)** Draw the straight line (D) passing through E and parallel to (d).
b) Choose any two points F and G on (D).
c) What is the distance from F to (d)? from G to (d)?

Result

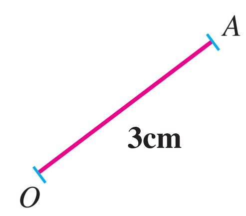


A variable point M lying at a constant distance ℓ from a given fixed straight line (d) describes two straight lines (D_1) and (D_2) parallel to (d) and lying at a distance ℓ from the given straight line.

2º) Circle

Activity

O and A are two fixed points such that $OA = 3$ cm.



1º) Place the points B, C and D such that :

$OB = OC = OD = 3$ cm.

2º) Draw the circle (C) of center O and of radius 3 cm.

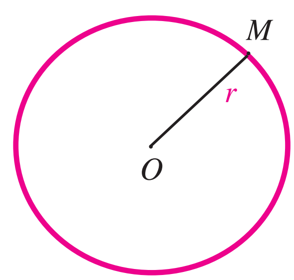
Are the points A, B, C and D on (C)?

3º) P and Q are any two points of circle (C).

Determine the lengths OP and OQ.

Result

A variable point M lying at a constant distance r from a fixed point O, describes a circle of center O and radius r .

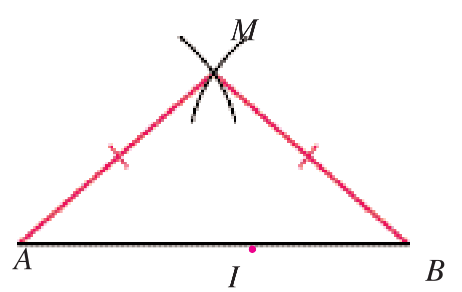


3º) Perpendicular bisector of a segment

Activity

[AB] is a segment having a length of 5 cm, and I is its midpoint.

From A and B as centers we draw two arcs of radius 3 cm which intersect at M.



1º) Measure MA and MB and complete:

$MA \ldots MB$

2º) From A and B as centers, draw from the other side of M with respect to [AB] two arcs of radius 4 cm that intersect at N.

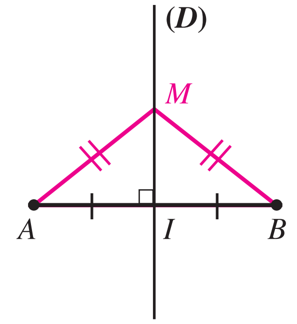
Complete : $NA \ldots NB$

3º) Are M, N and I collinear ? If yes, verify that the straight line (MN) is perpendicular at I to [AB].

4º) Choose any point of (MN). Measure PA and PB then complete : $PA \ldots PB$.

Result

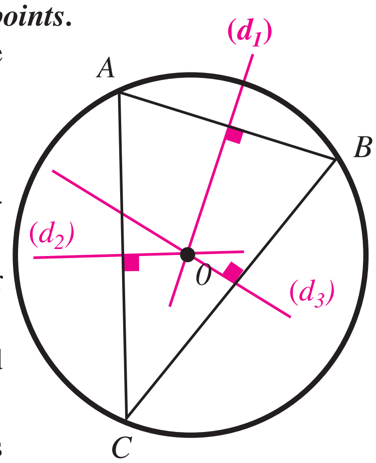
A variable point *M* equidistant from the extremities of a given segment moves on the perpendicular bisector of this segment .



Remark :

Circle passing through three non-collinear points.

Construct a circle (C) passing through three non-collinear points A, B and C .
The center O of circle (C) verifies
$OA = OB = OC$.
Since $OA = OB$, then O is on the perpendicular (d_1) of segment [AB].
Since $OA=OC$, then O is on the perpendicular bisector (d_2) of [AC].
Hence, O is the point of intersection of $(d_1)$and (d_2).
The perpendicular bisector (d_3) of [BC] should pass through O since $OB = OC$,
The center O of circle (C) passing through the three non-collinear points A, B and C is the point of intersection of two perpendicular bisectors of the segments [AB], [AC] and [BC].
(C) is called the **circumscribed circle** about triangle ABC.

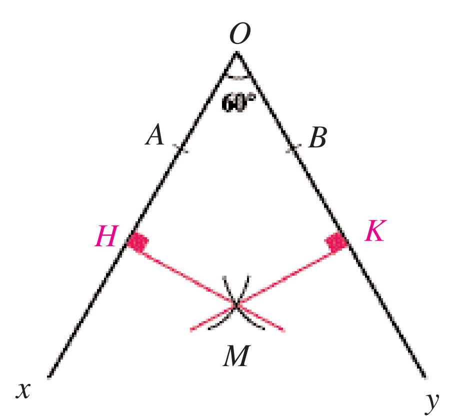


4°) Bisector of an angle

Activity

$\widehat{xOy}$ is an angle of measure 60°.
A and B are two points of [Ox) and [Oy) respectively such that $OA = OB = 2$ cm. From A and B as centers we draw two arcs of radius 3 cm which intersect at M.

MH and MK are the distances from M to [Ox) and to [Oy).



1°) Measure MH and MK and complete MH ... MK .

2°) Join O to M and measure each of the angles $\widehat{HOM}$ and $\widehat{KOM}$.

Complete : $\widehat{HOM}$ … $\widehat{KOM}$

3°) From A and B as centers, draw two arcs of radius 4 cm which intersect at N.

a) Is N a point of the straight line (OM)?

b) NE and NF are the distances from N to $[Ox)$ and $[Oy)$.

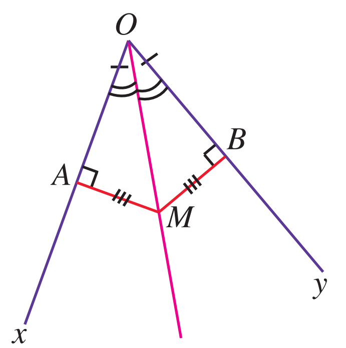
Measure NE and NF and then complete : NE … NF

4°) P is any point of the straight line (OM). PI and PJ are the distances from P to $[Ox)$ and to $[Oy)$.

Compare these two distances.

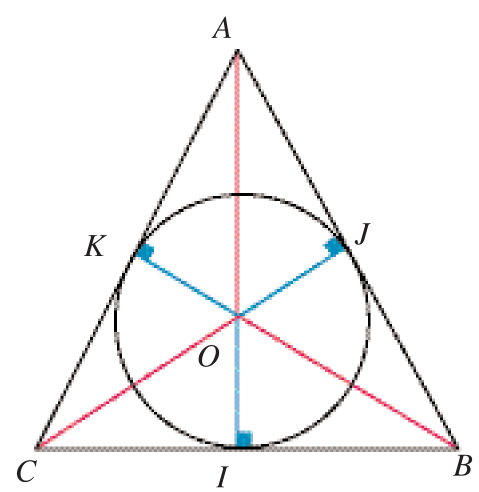
Result

A variable point *M* equidistant from the two sides of an angle moves on the bisector of this angle



Remark :

Inscribed circle in a triangle



Consider a triangle ABC.

The bisectors of angles $\widehat{BAC}$ and $\widehat{ABC}$ intersect at O.

O being on the bisector of $\widehat{BAC}$, is equidistant from $[AB]$ and $[AC]$, therefore $OK = OJ$.

O being on the bisector of $\widehat{ABC}$, is equidistant from $[BA]$ and $[BC]$, therefore $OI = OJ$.

Hence $OK = OJ = OI$ and O is thus, the center of the circle of radius $OK = OI = OJ$.

This circle is called **inscribed circle** of triangle ABC.

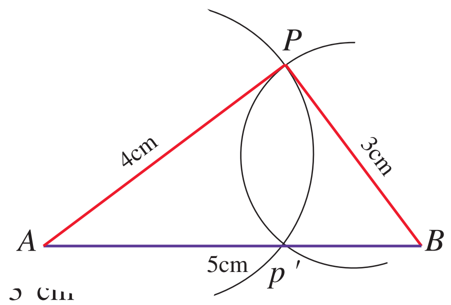
Since $OI = OJ$, then O is on the bisector of angle $\widehat{ACB}$.

The center O of the circle inscribed in triangle ABC is the point of intersection of the two bisectors of the angles of this triangle.

3 CONSTRUCTIONS

EXAMPLE 1

Let [AB] be a segment of length 5 cm. Locate a point P which is 4 cm from A and 3 cm from B.



- P being 4 cm from A, is on the circle of center A and radius 4 cm.
- P being 3 cm from B, is on the circle of center B and radius 3 cm.
  We draw arcs of centers A and B and radii 4 cm and 3 cm respectively. The point of intersection of the two arcs is the point P :

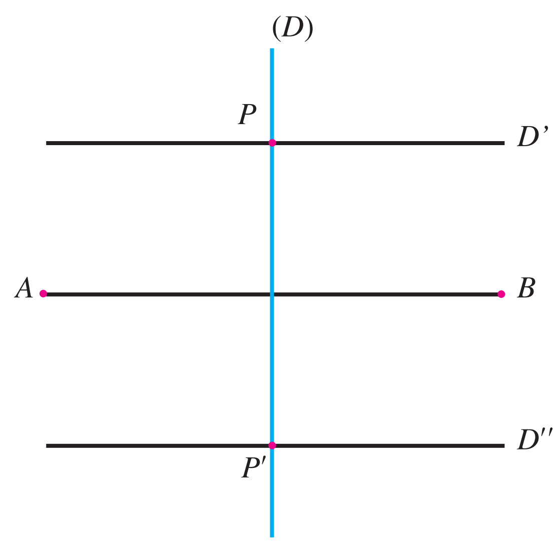
  $PA = 4$ cm and $PB = 3$ cm.

Remark :

There exists a second point P' such that $P'A = 4$ cm and $P'B = 3$ cm, lying on the other side of [AB] and which is the second point of intersection of the two circles.

EXAMPLE 2

Let [AB] be a segment of length 6 cm. Locate a point P equidistant from A and B and which is at a distance of 4 cm from the straight line (AB).



- P being equidistant from A and B, is on the perpendicular bisector (D) of [AB].
- P being at a distance of 4 cm from the straight line (AB), is on a parallel line (D') to (AB), lying at 4 cm from (AB).

The point of intersection of (D) and (D') is the required point P .

Remark :

There exists a second point P' which is the point of intersection of (D) and the parallel (D'') to (AB) lying on the other side of (AB) and at 4 cm from (AB).

Exercises and problems

For testing the knowledge

1 *ABCD* is a fixed rectangle, *I* and *J* are the midpoints of [*AD*] and [*BC*] respectively. (*D*) is a variable straight line perpendicular to [*AB*]. (*D*) cuts [*AB*], [*IJ*] and [*DC*] at *M*, *H* and *N* respectively.

1º) What are the fixed points of the figure ? The variable points ?

2º) On which fixed line does point *H* move ?

2 (D_1) and (D_2) are two fixed parallel straight lines at a distance of 4 cm from each other. A variable straight line (*d*) perpendicular to (D_1) cuts respectively (D_1) and (D_2) at *I* and *J*.
On which fixed line does the midpoint *O* of [*IJ*] move ?

3 *RAT* is a triangle such that *A* and *T* are fixed. The height from *R* cuts the straight line (A*T*) at *O* with *RO* = 5 cm. Find the fixed line on which point *R* is moving.

4 What is the fixed line described by the extremity of a clock’s minute hand of diameter 2.8 cm ?

5 *A* being a fixed point, what is the fixed line described by the centers *O* of circles of radii 3 cm and passing through *A*?

6 *ABC* is an isosceles triangle of vertex *A* such that *AB* = *AC* = 4 cm. If *A* and *B* are fixed and *C* is variable, find the fixed line described by point *C*.

7 *LOI* is a triangle such that *O* and *I* are fixed. The median drawn from *L* cuts [*OI*] at *M*. Find the line on which the variable point *L* moves if *LO* = *LM*.

8 $\widehat{xAy}$ is an angle which measures 80° and has a fixed vertex A . Determine and construct the line described by the point O that is equidistant from $[Ax)$ and $[Ay)$.

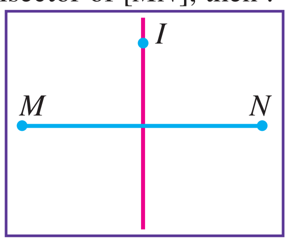
9 Choose the correct answer.

1°) I being a point of the perpendicular bisector of $[MN]$, then :

$IM < IN$ ☐

$IM = IN$ ☐

$IM > IN$ ☐

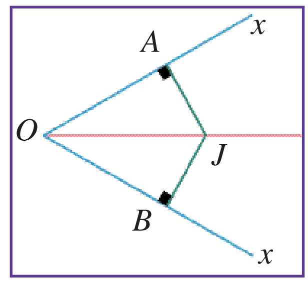


2°) J being a point of the bisector of the angle $\widehat{xOy}$, then :

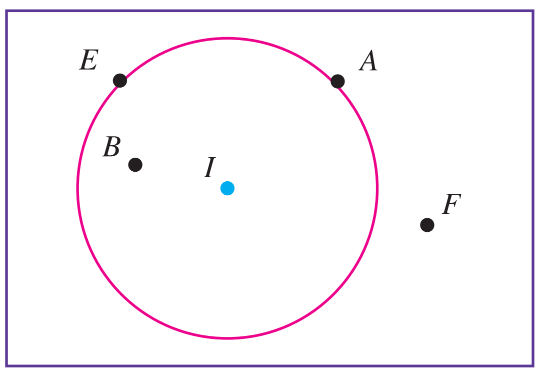
$JA = JB$ ☐

$JA < JB$ ☐

$JA > JB$ ☐



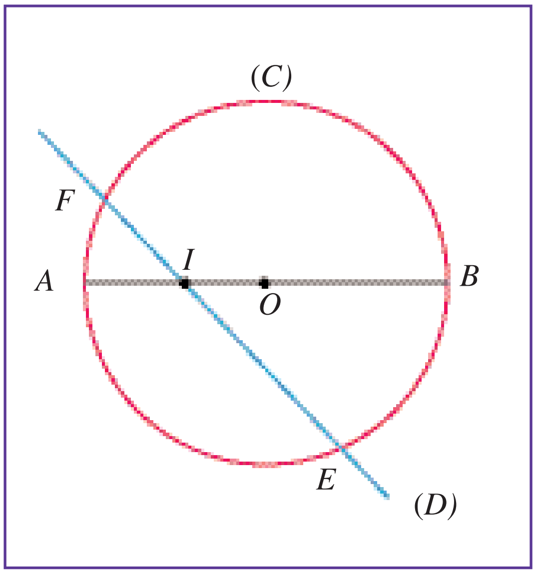
3°) By observing the figure below, representing a circle of center I, we can write :



$IA = IE$ ☐ $IA > IF$ ☐ $IB < IE$ ☐ $B > IF$ ☐

10 Answer by true or false.

(C) is the circle of center O and of fixed diameter [AB]. A variable straight line (D) cuts the circle at E and F and [AB] at I.



1º) A and B are variable points.

2º) The points E and F are fixed.

3º) The point I is variable.

4º) OA is constant.

5º) OI is constant.

6º) EF is variable

7º) OE is constant.

8º) I describes the diameter [AB].

9º) E moves on the straight line (D).

10º) F describes the circle (C).

11º) Every point of circle (C) is equidistant from A and B.

12º) Every point of circle (C) is equidistant from the sides of angle $\widehat{EIB}$.

13º) O is on the perpendicular bisector of [AB].

For seeking

11 Two straight lines (AB) and (CD) intersect at I.

a) On which fixed line are lying the points that are at 3 cm from (AB)?

b) On which fixed line are lying the points that are at 4 cm from (CD)?

c) Deduce how many points lie at the same time at 3 cm from (AB) and at 4 cm from (CD)?

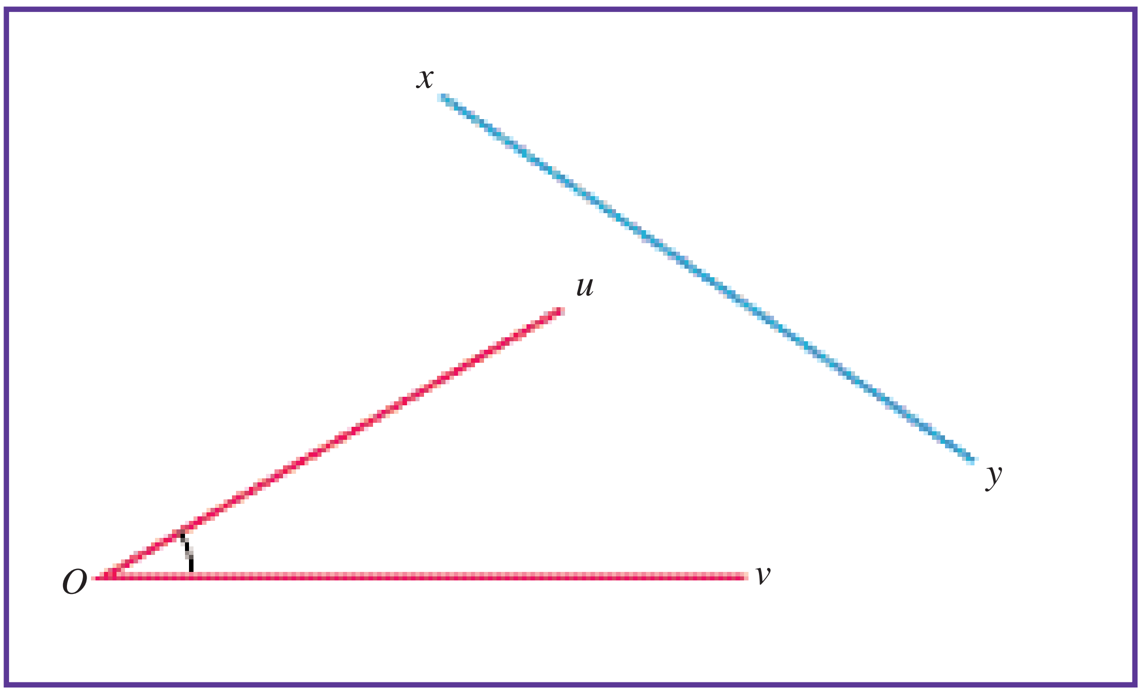
12 (xy) and (uv) are two parallel straight lines at a distance of 6 cm from each other.

a) On which fixed line are found the points equidistant from (xy) and (uv) ?

b) P is a point lying at 1 cm from (xy) and between the parallels (xy) and (uv).
Construct the points lying at 2 cm from P and equidistant from (xy) and (uv).

13 $[AB]$ is a segment of length 6 cm. Construct two points C and D lying at 4 cm from A and at 5 cm from B respectively.

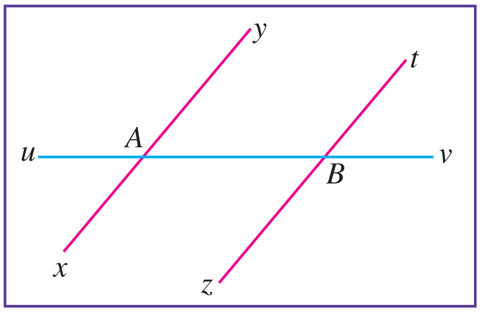
14 Construct a point I lying on (xy) and equidistant from the sides $[Ou)$ and $[Ov)$ of angle $\widehat{uOv}$.



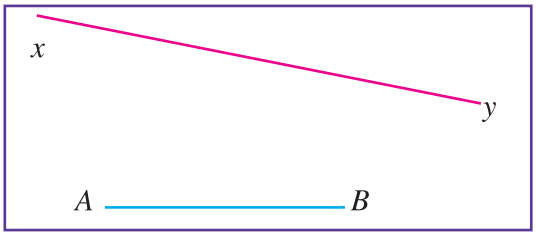
15 In the adjacent figure, (xy) and (zt) are parallel, cut by (uv) at A and B respectively.

1º) Draw the bisectors $[At)$ and $[By)$ of angles $\widehat{yAB}$ and $\widehat{tBA}$ respectively. Let I be the point of intersection of the two bisectors.

2º) Show that I is equidistant from the three straight lines (xy), (uv) and (zt) .



16 Construct point I lying on (xy) and equidistant from A and B.



17 **a)** Construct triangle ABC such that $AB = 6$ cm, $AC = 5$ cm and $BC = 4$ cm. How many solutions are there ?

b) Can we construct triangle ABC such that $AB = 6$cm , $AC = 2$ cm and $BC = 3$ cm?

18 Let H be a variable point on the semi-straight line $[Ox)$. On the perpendicular at H to $[Ox)$ we take two points M_1 and M_2 such that $HM_1 = HO = HM_2$.

a) Calculate each of the angles $\widehat{M_1OH}$ and $\widehat{M_2OH}$.

b) Find and construct the fixed line described by the points M_1 and M_2 when H varies on $[Ox)$.

TEST

1 Complete the following table : (M is a variable point of the plane). **(5 points)**

Property of M	Fixed line described by M
M is equidistant from two fixed points A and B.	
	M is on the bisector $[Ou)$ of angle $\widehat{xOy}$.
	M is on the circle of fixed center O and radius r.
M is at a constant distance d from a fixed straight line (D).	
M is equidistant from two parallel straight lines (D_1) and (D_2).	

2 **a)** Draw a segment $[AB]$ of length 8 cm.

b) From a point O as center, not belonging to (AB), draw a circle, with a convenient radius, which cuts (AB) at C and D.

c) The bisector $[Ou]$ of $\widehat{COD}$ cuts (AB) at E. Verify that E is the midpoint of $[CD]$ and that (OE) is perpendicular to (AB). **(4 points)**

3 **a)** Construct triangle ABC such that $AB = 6$ cm, $BC = 7$ cm and $CA = 8$ cm. **(2 points)**

b) Find and construct the point O equidistant from the three sides $[AB]$, $[AC]$ and $[BC]$ of triangle ABC. **(2 points)**

4 Construct triangle ABC such that $BC = 6$ cm. Locate point A such that $AB = 5$ cm and the distance between A and (BC) is 4 cm. How many solutions are there ? **(4 points)**

5 Let [AB] be a fixed segment of length 6 cm. $[Ax)$ is a variable semi-straight line and C is the symmetric of B with respect to $[Ax)$. Find and construct the fixed line described by C when $[Ax)$ varies . **(3 points)**

24 SPACE GEOMETRY

Objectives

- Constructing a right rectangular prism, a cube and a right prism by preparing the model of each.
- Drawing a right rectangular prism in perspective.
- Drawing a right prism in perspective.
- Calculating the lateral area and the total area of a cube, a right prism.
- Calculate the volume of a cube, a right rectangular prism and a right prism.

CHAPTER PLAN

COURSE

EXERCISES AND PROBLEMS

TEST

PERSPECTIVE

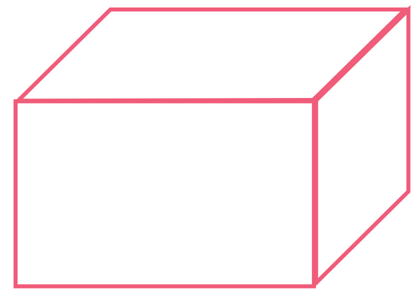


Figure 1

The figure 1 is a photograph of a chalk box.

The figure 2 is a representation of this box in «perspective».

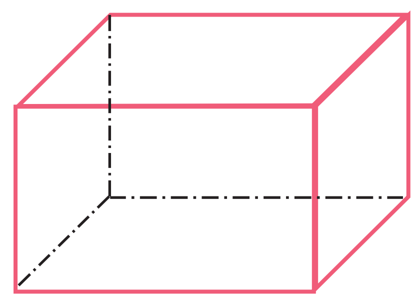


Figure 2

We do drawings in «perspective» to represent solids on a paper.

In a perspective :

- two parallel straight lines, in reality, are represented by two parallel straight lines ;
- two parallel segments of equal length are represented by two parallel segments of equal length ;
- an invisible straight line (hidden) is drawn dotted ;
- the right angles of real objects are not all represented by right angles ;
- the rectangles are often represented by parallelograms.

The adjacent figure represents a solid in perspective.

Name the parallel edges.

Name the hidden edges.

Name the hidden parallelograms.

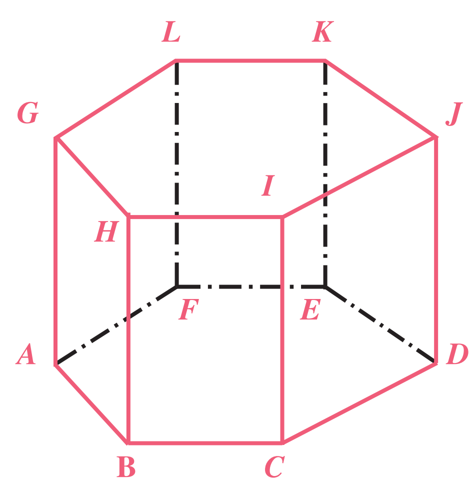


Figure 3

PRELIMINARY ACTIVITY : RIGHT RECTANGULAR PRISM

Activity

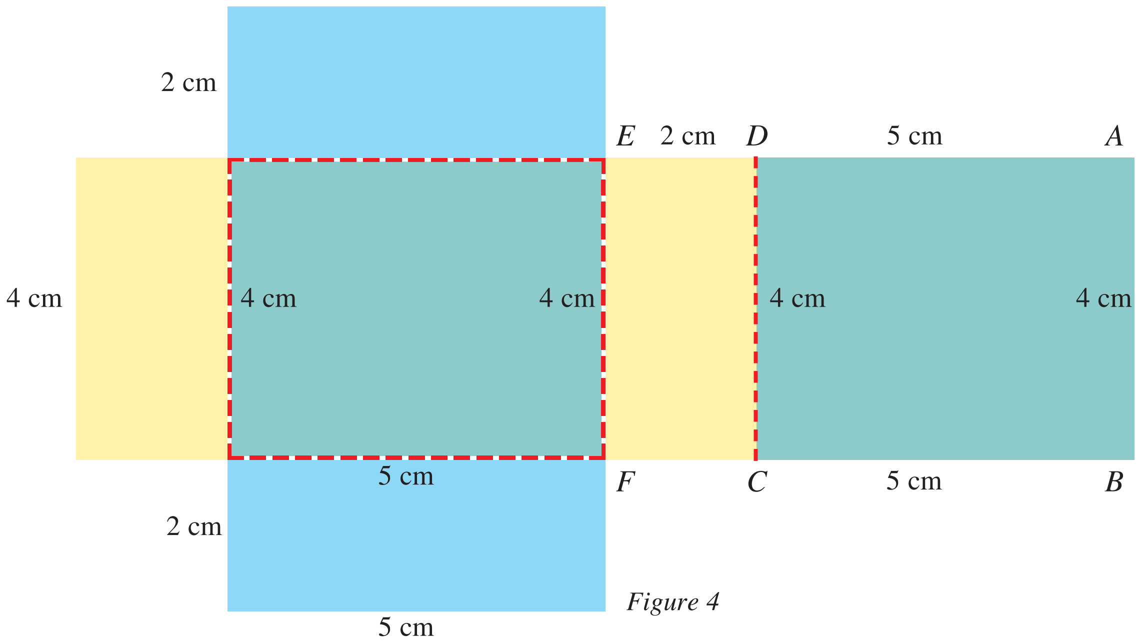


Figure 4

Copy this model on a paper.
Fold it according to the dotted line and assemble the solid by using adhesive ribbon.
Place the points D, E and F on the obtained solid.
How many rectangles do you need to construct this solid ?

A rectangular parallepiped or a a right rectangular prism is a solid which has six rectangular faces.

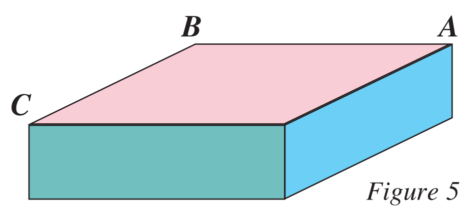


Figure 5

Each side of a rectangle is called an edge.
The points A, B, C, D, E, F, G, H are the vertices.
In a right rectangular prism there are 8 vertices and 12 edges.
The edges issued from the same vertex are perpendicular.
The parallel edges have the same length.
The lengths of the three edges issued from the same vertex are called the dimensions.

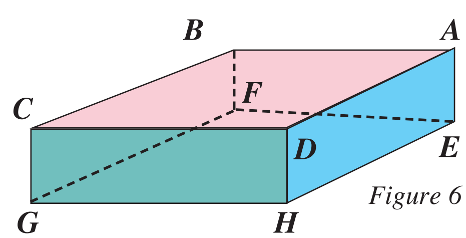


Figure 6

Cube

Activity

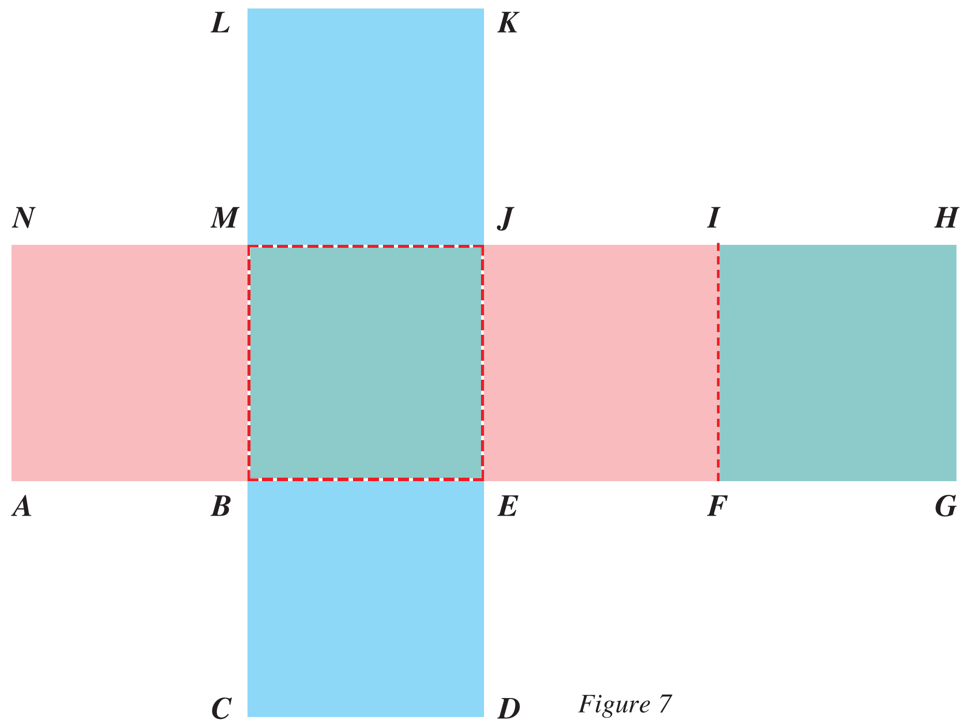


Figure 7

Figure 7 represents the model of a right prism.

How many squares does it contain ? Name them.

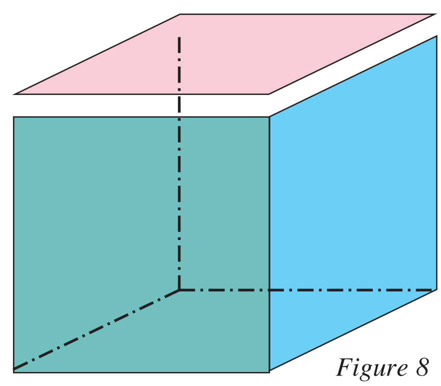


Figure 8

A cube is a right prism where the six faces are squares (fig 8).

Application

Figure 9 represents a right prism.

Name the right angles of vertex A of figure 9.

Which rectangle is congruent to rectangle $ADEF$?

If the edge $[AD]$ measures 3 cm, what is the length of the edge $[HG]$?

If the two faces $ABGF$ and $ADEF$ are squares, would figure 9 represent a cube?

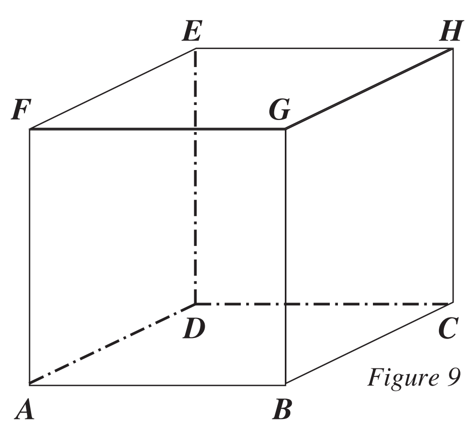


Figure 9

2 PRELIMINARY ACTIVITY : RIGHT PRISM

Activity

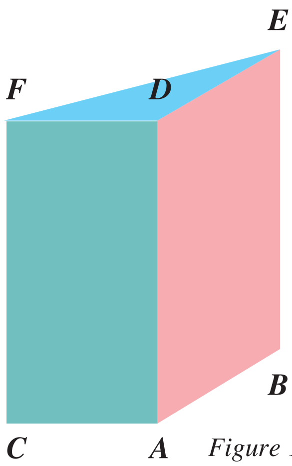


Figure 11

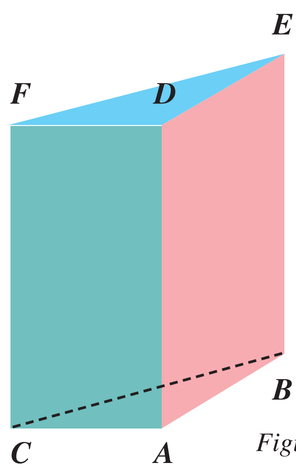


Figure 12

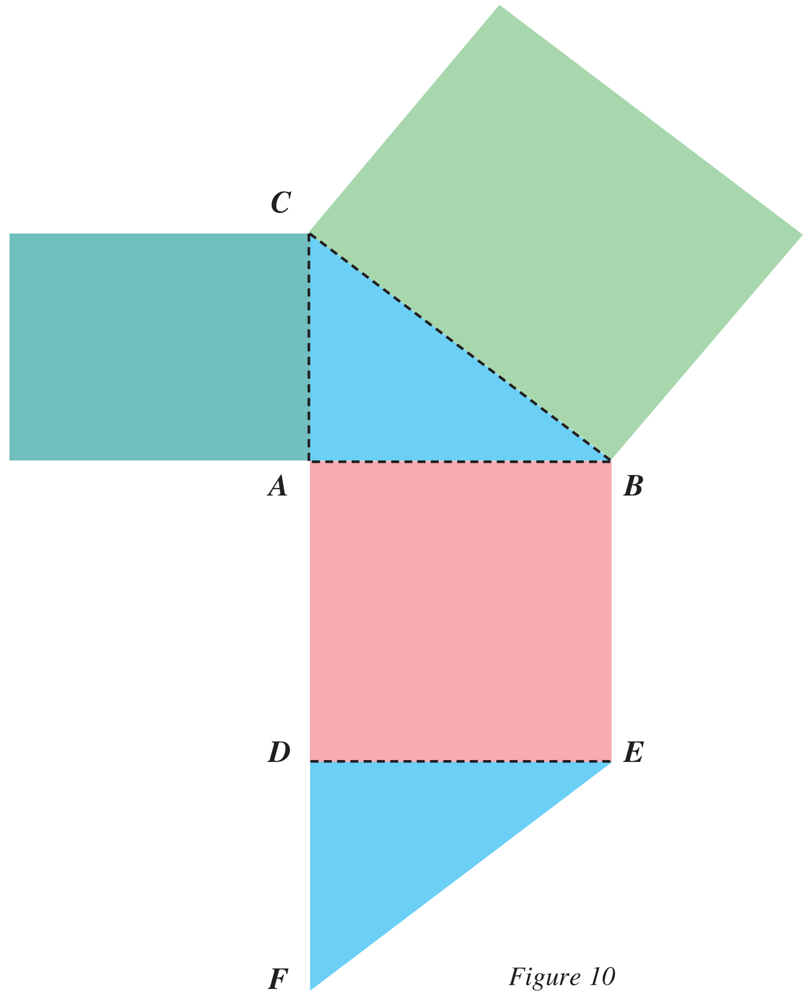


Figure 10

Copy the model of figure 10 on a paper.
Fold it according to the dotted line and assemble the solid by using an adhesive ribbon.
Figure 11 represents this solid.
Figure 12 represents this solid in prespective.

A right prism is a solid which has two congruent bases (*ABC* and *DEF*) and rectangular faces (*ABED*, *BCFE* and *CFDA*).

The edges of a right prism, which are not the sides of the bases, are the heights of the prism. Each of the lengths *AD*, *BE* and *CF* is the height of the prism of figure 12.
The base of a right prism is a polygon (triangle, quadrilateral,...).
The sides of the faces are the edges of this prism.

Application

Is every right rectangular prism a right prism ?

Figure 13 represents a cube.

Can you consider the square *ABCD* as a base of a prism of height *AE* ?

If *ADHE* is the base of the prism, what is then its height ?

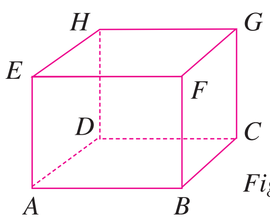


Figure 13

Remark :

Every right rectangular prism is a right prism having a rectangle as base.

Application

Figure 14 represents a right prism.

Name the bases.

What is the nature of the base ?

Name the faces.

What is the height ?

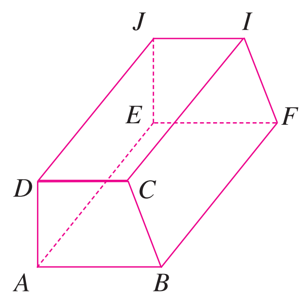


Figure 14

3 LATERAL AND TOTAL AREA OF A RIGHT RECTANGULAR PRISM AND OF A RIGHT PRISM

Here is the model of a right prism of triangular bases.

Measure in cm the lengths of [*BI*], [*BF*], [*IF*] and [*FG*].

Calculate in cm^2 the sum of the area of the rectangles.

Calculate the product $(BI+IF+FB)\times IJ$.

Compare the area of the rectangles to the product just found.

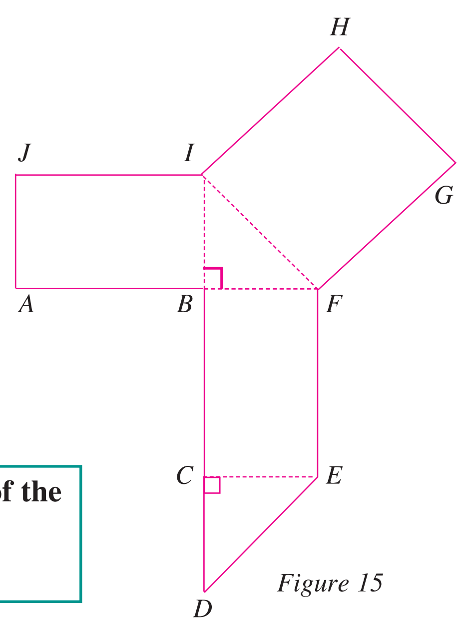


Figure 15

We call lateral area of a right prism the sum of the areas of the faces.

The lateral A_L of a right prism is equal to the product of the perimeter of the base by the height.
A_L = perimeter of the base × height

The sum of the lateral area and of the areas of the bases of a right prism is called the total area of this prism.
$A_t = A_L$ + area of the two bases.

Application

1- Calculate the total area of a cube when the edge measures 8cm.

2- Calculate the total area of a right rectangular prism of dimensions 3cm, 4cm and 5cm.

4 VOLUME OF A RIGHT RECTANGULAR PRISM AND OF A RIGHT PRISM

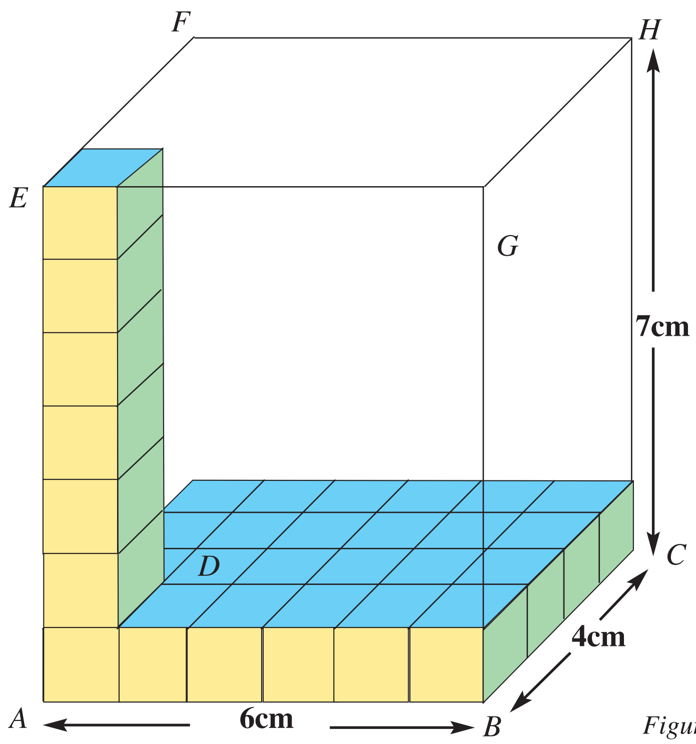
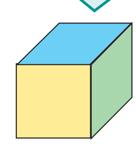


Figure 16

A cube of edge 1 cm .



The drawing above represents a right rectangular prism of dimensions 4cm, 6cm and 7 cm.
a) How many cubes of 1 cm edge should we have to cover the base *ABCD* of this rectangular prism ?
b) How many cubes of 1 cm edge can we place along the height [*AE*]?
c) What is the number of cubes of edge 1 cm that is necessary to fill exactly this prism ? Is it the number $7 \times 6 \times 4$?

The volume of a right rectangular prism is equal to the product of its three dimensions.

Application

Calculate the volume of a right rectangular prism of dimensions 3m, 4m and 5 m.

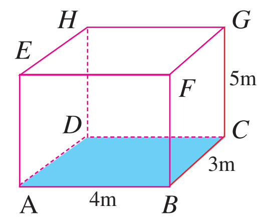


Figure 17

The rectangle *ABCD is often called the* base of the rectangular prism and *BF* the height.

If $AB > AD$, we call $AB = L$ the length of the base, $AD = l$ the width and $BF = h$ the height.

$B = L \times l$ is the area of the base.

V is the volume,

then $V = B \times h = L \times l \times h$.

The volume V of a right prism is equal to the product of the area B of the base by the height h.

$$\boldsymbol{V = B \times h}$$

Application

Figure 18 represents a right prism.

We suppose that the base *IJK* is a right triangle at *I* such that

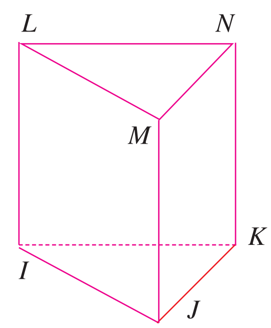
$IJ = 3$cm and $IK = 4$cm.

Calculate the area B of the base of this prism.

Calculate the volume V of this prism if

$IL = 5$ cm

Figure 18



EXERCISES AND PROBLEMS

For testing the knowledge

1 Answer by true or false.

1º) A parallepiped is not a right prism.

2º) A right rectangular prism has as many edges as vertices.

3º) A right rectangular prism can have two triangular faces.

4º) A cube is not a right prism.

5º) The total area of a cube is equal to six times the area of a face.

6º) The area of a rectangular prism is always equal to six times the area of each face.

7º) The volume of a cube is equal to the length of the edge cubed.

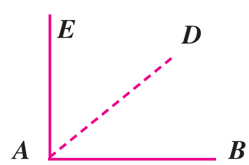
8º) Every cube is a right prism.

9º) The faces of a right prism are rectangles.

10º) In perspective, a rectangle is always represented by a rectangle.

11º) A right prism of triangular bases can have five faces.

2 Draw in perspective a right rectangular prism of edges [*AB*], [*AE*], [*AD*].



3 A box having the form of a rectangular prism has : a length $L = 5$ dm, a width $\ell = 3$ dm and a height $h = 2.5$ dm.

a) Give L, ℓ and h in cm.

b) Calculate the volume of this box in cm^3, then in dm^3 .

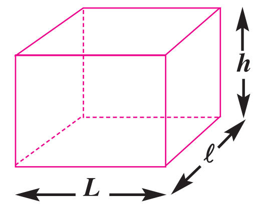
4 The total area of a cube is $54m^2$. What is the length of its edge ?

5 What is the length of an edge of a cube of volume $27m^3$?

6 The lateral area of a right prism, of height 6cm, is equal to 72 cm^2 and its base is an equilateral triangle. What is the length of the side of the base ?

7 L, ℓ, h, A and V are respectively the length, the width, the area of the base and the volume of a rectangular prism. Complete this table.

L (in cm)	ℓ (in cm)	h (in cm)	A (in cm^2)	V (in cm^3)
22	14	5		
18		6	90	
15		8		1320
20			260	2080

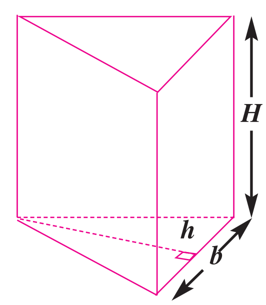


8 Complete this table.

A is the area of the base,

V is the volume,

b (in cm)	h (in cm)	H (in cm)	A (in cm^2)	V (in cm^3)
5	2	8	12 6	
6		5		
4				12
	4	10		50



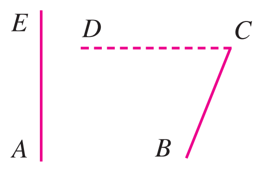
a right prism

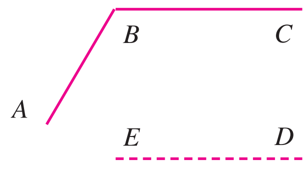
9 Calculate the volume of a rectangular prism whose base is a square and whose height measures 100 cm and its width is 30 cm less than its height.

10 Calculate the volume of a rectangular prism whose length measures 60 cm and whose height is equal to one third the length and half the width.

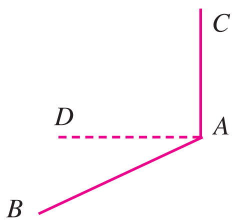
For seeking

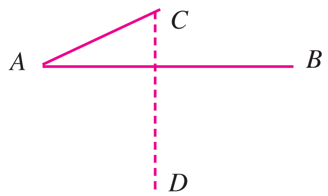
11 Complete, in perspective, the drawings of the right prisms.





12 Complete, in perspective, the drawings of right prisms having a triangular base.





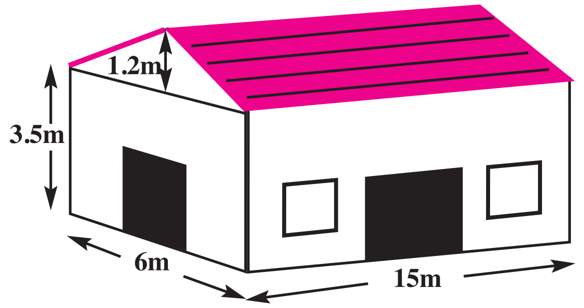
13 A camping tent of 2.40m length, 1.30m width and 1.20m height has the form of a right triangular prism.

Calculate the area of the carpet of the ground of the tent.

Calculate the volume available under the tent.

How many liters of air does this tent contain ? ($1dm^3 = 1$ liter)

14 Calculate the volume of this house.



TEST

1 *ABCDEFGH* represents a right rectangular prism.

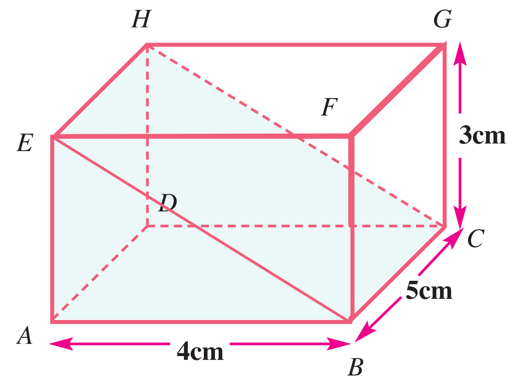
a) Calculate the total area of this prism. **(3 points)**

b) Use a ruler to measure [*BE*]. **(2 points)**

c) Calculate the area of the base of the yellow prism. **(2 points)**

d) Calculate the volume of the yellow prism. **(2 points)**

e) Calculate the total area of the yellow prism. **(3 points)**



2 A right prism has as base an equilateral triangle *ABC* such that $AB = 5$ cm and as face a square *ABDE*. Calculate its lateral area. **(4 points)**

3 A right rectangular prism has as dimensions $AB = 5$ cm, $AD = 2AB$ and $AE = AB + AD$.

a) Complete, below, the diagram of this rectangular prism in perspective.

b) Calculate the total area and the volume of this rectangular prism. **(2 points)**

c) Calculate the volume of the right prism having as base the triangle *ABD* and as height [*AE*]. **(2 points)**