المادة : رياضيات
المدة : ساعتان

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I.

1) The curve (C) below is the graphical representation of a function g defined over ℝ by: $g(x) = (ax + b)\, e^x + c$, where a, b and c are real numbers.

a) Show that $g(x) = (x - 2)\, e^x + 2$.

b) Show that the equation $g(x) = 0$ admits in the interval [1; 2] a unique solution α.

c) Prove that g admits an inverse function over]1;+∞[and trace its curve (C').

d) Calculate the area of the domain limited by (C'), th
= 2 and the two straight lines of equations y = 1 and

2) Let f be the function defined over ℝ* by $f(x) = \frac{e^x - 1}{x^2}$.

Designate by (Γ) its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

a) Determine $\lim_{x \to 0^-} f(x)$ and $\lim_{x \to -\infty} f(x)$.

b) Prove that $f'(x) = \frac{g'(x)}{x^3}$, and construct the table of variations of f.

c) Given that $\alpha = 1.75$ give an approximate value of $f(\alpha)$ to the nearest 10^{-2},then trace (Γ).

II.

$(O; \vec{u}; \vec{v})$ Is an orthonormal syste

1) Locate the points A, B and D of Z_B=2 and Z_D= -2 + 2i.

2) Calculate the affix of point C when ABCD is a parallelogram and locate C.

3) E is a vertex of the right isosceles triangle CBE with $(\overrightarrow{BC}, \overrightarrow{BE}) = -\frac{\pi}{2}\ (2\pi)$.

Show that $\frac{z_E - z_B}{z_C - z_B} = -i$ and deduce the affix of E.

4) Calculate the affix of point F if CDF is right isosceles at D with $(\overrightarrow{DC}, \overrightarrow{DF}) = \frac{\pi}{2}\ (2\pi)$.

5) Show that: $\frac{z_F - z_A}{z_E - z_A} = -i$.

Deduce the nature of triangle AEF.

III. For an exam each of ten teachers prepares two questions. The 20 questions are placed in 20 identical envelopes. Two candidates sit for the exam; each one chooses, at random, two questions.

The questions chosen by the first candidate are not offered for the second.

Denote by A_1 the event: the two questions obtained by the first candidate are written by the same teacher.

Denote by A_2 the event: the two questions obtained by the second candidate are written by the same teacher.

1) Show that $P(A_1) = \frac{1}{19}$.

2)
   a) Calculate $P(A_2/A_1)$
   b) Calculate the probability that each of the two candidates get two questions written by the same teacher.

3)
   a) Calculate $P(A_2/\overline{A_1})$.
   b) Deduce $P(A_2)$ and $P(A_1 \cup A_2)$

4) Let X be the random variable equal to the number of candidates that have chosen two questions written by the same teacher.
   Determine the probability distribution of X.

IV.

In the space of an orthonormal system $(O;\vec{i};\vec{j};\vec{k})$, consider the points A(3;1;0), B(1;3;0), C(3;2;1) and D(0;0;2).

1) Show that ABCD is a tetrahedron, and calculate its volume.

2)
   a) Calculate the components of the vector $\vec{n} = \overrightarrow{AB} \wedge \overrightarrow{AC}$.
   b) Deduce the area of triangle ABC, and also find the distance from D to the plane ABC.

3)
   a) Find an equation of plane ABC.
   b) Verify that, an equation of the plane (ABD) is $x + y + 2z - 4 = 0$.
   c) Find a system of parametric equations of the intersection of the planes (ABC) and (ABD).

4)
   a) Show that the point D is equidistant from A, B and C.
   b) Find a system of parametric equations of the axis (d) of the circle (C) circumscribed about triangle ABC.
   c) Find the coordinates of the center ω of (C) and calculate its radius.
   d) Find a system of parametric equations of the tangent (T) drawn from A to (C).