Pressure

1- Notion of Pressure

On snow, a non-equipped walking man leaves deep foot prints; if he uses the skis, the prints are less deep. We say that pressure on snow becomes less.

- On what factors does the pressure depend?
- How can we show evidence of the influence of these factors?

2- Definition of Pressure on a plane surface:

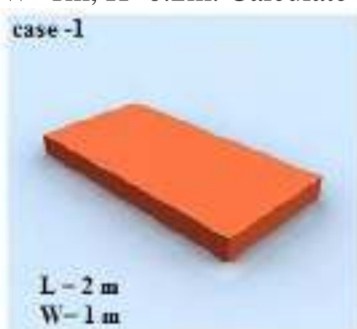
The force $\vec{F}$ of magnitude F applied perpendicularly and uniformly on a plane surface (S), the **pressure** exerted is the force acting on a unit area. So The pressure expressed in Pascal's **(Pa)** and defined as the ratio of the magnitude of the pressing force **F** to the area of the surface of contact **S**:

$$P = \frac{F}{S}$$

P= pressure in **(Pa)**
F= force applied in (N)
S= contact area in **(m^2)**

Example-1:

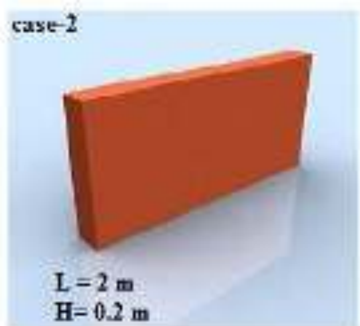
Consider a rectangular brick have a weight 300N have the following dimensions: L=2m, W=1m, H=0.2m. Calculate the pressure exerted by brick for its three different faces.



$S = 2\,m \times 1\,m = 2\,m^2$

$F = 300\,N$

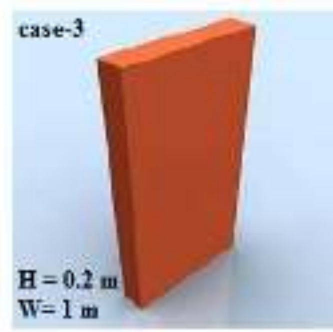
$P = \frac{300}{2} = 150\,Pa$



$S = 2\,m \times 0.2\,m = 0.4\,m^2$

$F = 300\,N$

$P = \frac{300}{0.4} = 750\,Pa$



$S = 1\,m \times 0.2\,m = 0.2\,m^2$

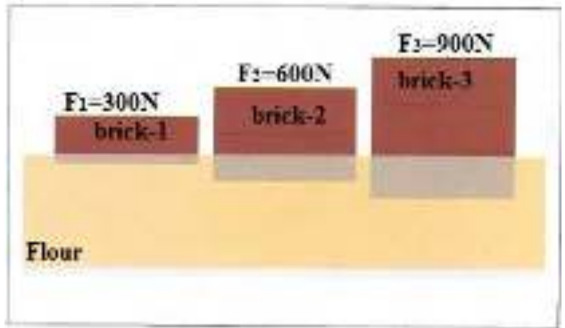
$F = 300\,N$

$P = \frac{300}{0.2} = 1500\,Pa$

- **We notice that the face that has smallest surface area have the highest pressure so pressure inversely proportion to surface area at a constant force.**

Example-2:

Consider a three rectangular brick have different weights 300N, 600N, 900N have the same surface area **S= 2 m²** are put on flour. Calculate the pressure exerted by brick for its three different faces.



- **For brick -1**

  $P_1 = \frac{F1}{S} = \frac{300}{2} = 150$ pa.

- **For brick -2**

  $P_2 = \frac{F2}{S} = \frac{600}{2} = 300$ pa.

- **For brick -3**

  $P_3 = \frac{F3}{S} = \frac{900}{2} = 450$ pa.

- **We notice that the brick that has highest weight force (brick-3) a have the highest pressure and highest depth in flour, so pressure proportion to pressing force at a constant** surface area**.**

Revision:

- **Density:**

  $\rho = \frac{m}{V}$ Or → $m = V \times \rho$ → $V = \frac{m}{\rho}$

  ρ = density (kg/m³)
  M= mass (Kg)
  V=volume (m³)

  - $\rho_{water} = 1000 kg/m^3 = 1 g/cm^3$
  - $(g/cm^3) \xrightarrow{x1000} (kg/m^3)$

- **Volume:**

  $V = S \times H$

  H=height(m)
  S=surface area (m²)
  V=volume(m³)

- **Mass:**

  $M = V \times \rho$ **and** $V = S \times H \Rightarrow m = S \times H \times \rho$

- **Rule of some surface area:**
  - square $S = a^2$
  - rectangle $S = L \times W$
  - Circle $S = \pi r^2$

3- Pressure in Liquids.

3.1- Liquid exerts a pressure on all bodies immersed in it. The pressure due to the liquid $\mathbf{P_A}$ at a point A at height **h** in a liquid at rest, and of density **ρ** is given by following expression:

$$\mathbf{P = \rho .\ g.\ h}$$

3.2- Derivation of expression is as follow:

$$P = \frac{F}{S} = \frac{m.g}{S} = \frac{\rho.V.g}{S} = \frac{\rho.S.h.g}{S} \quad \Rightarrow \quad \mathbf{P = \rho.\ g.\ h}$$

$\mathbf{P = \rho.\ g.h}$ — Or → $\rho = \frac{P}{g.h}$ → $h = \frac{P}{\rho.g}$

p= pressure at point A in (pa)
ρ = density of liquid in (kg/m^3)
g = gravity in (N/ kg)
h = height of liquid above point A in (m)

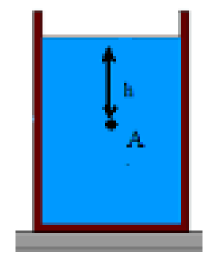
Note: pressure of liquid can be measure by manometric gauge

3.3- Pressure is proportion to:

a. As ρ increase, where g and h constant, P increase (density proportion to pressure)
b. As h increase, where g and ρ constant, P increase. (height proportion to pressure)

<u>Example:</u>

Calculate the pressure due to liquid, at point A inside the closed container.



Given: ρ_{water}= 1g/cm^3, g= 10N/kg, h_A= 15cm.

Given: ρ_{water}= 1g/cm^3=1000kg/m^3

h_A= 15cm=15/100=0.15m.

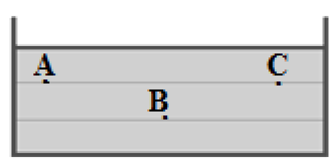
P= ρ. g. h=1000x10x0.15=1500pa

3.4- Pressure at two point in liquid:

In the same liquid and at same horizontal level pressure is the same.

<u>Example-1:</u>

Determine at which points the pressure is the same or different.

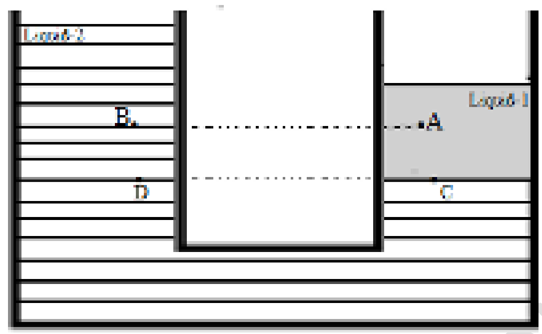


- The two point A and C are in **same horizontal level and in same liquid** so at these two-point pressure is the same (P_A=P_C)
- The two point A and B are not in same horizontal level so at these two-point pressure is different ($P_A \neq P_B$).

Example -2:

Consider a two different liquid in a U tube:

Determine at which points the pressure is the same or different.



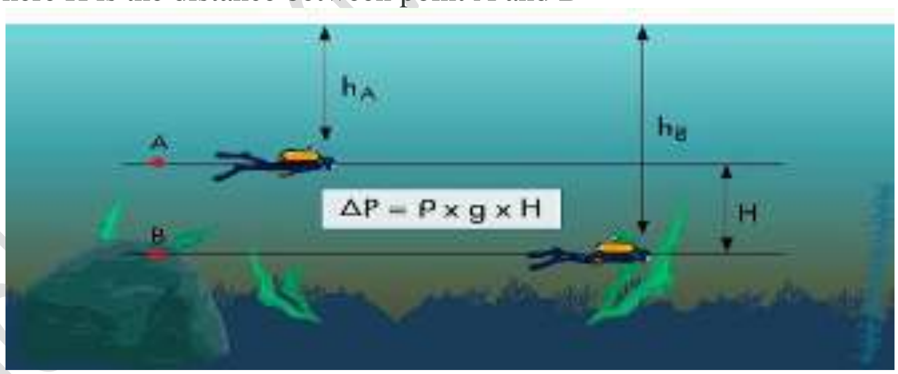
- The two-point C and D are in same horizontal level and in same liquid so the at these two-point pressure is the same ($P_C = P_D$)
- The two-point A and B are in same horizontal level but in different liquids so the at these two-point pressure is different ($P_A \neq P_B$).

3.5- Principle of hydrostatic

The difference of pressure between two points A and B in a liquid at rest is given by the fundamental principle of hydrostatic.

$$\Delta P = P_B - P_A = \rho \times g \times (h_B - h_A) = \rho \times g \times H$$

Where H is the distance between point A and B

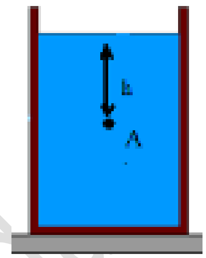


4- Atmospheric pressure:

The only pressure present on surface of a liquid is atmospheric pressure, and the instrument used to measure atmospheric pressure is **Barometer**. knowing that P_{atm} equivalent to pressure of mercury of height 76 cm. ($P_{atm} = P_{Hg}$ of height 76 cm.) so **P_{atm}=103360pa**

5- Total pressure:

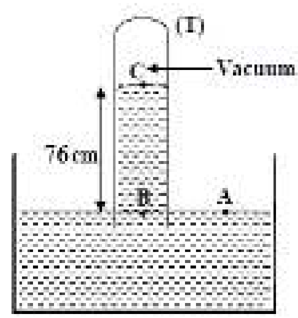
The pressure $\mathbf{P_A}$ exerted from liquid at point A and also there is a pressure on surface of a liquid that it is contact with air which is atmospheric pressure $\mathbf{P_{atm}}$**,** so total pressure $\mathbf{P_t}$ at point A it given by the following expression:



$$\mathbf{P_t = P_A + P_{atm}}$$

<u>Example-1:</u>

A group filled the tube (T) completely with mercury of density $\rho_{Hg} = 13600\ kg/m^3$, then turned it upside down and immersed it in a container containing mercury. The level of the mercury dropped down and settled at 76 cm above the free surface of the mercury that is found in the container. Given g = 10 N/kg



1- What is the value of the pressure P_C at C? Why?
2- Determine, in Pascal, the value of the pressure P_{Hg} exerted by mercury at B.
3- Determine, in Pascal, the value of the total pressure P_B at B.
4- The pressure at A and the pressure at B have the same value. Why?
5- Deduce the value of the atmospheric pressure P_{atm}.

<u>solution:</u>

given: h=76cm=0.76m, $\boldsymbol{\rho}_{Hg} = 13600\ kg/m^3$, g = 10 N/kg.

1- P_C= 0pa, since above point C there is vacuum and vacuum don't exert any pressure.
2- $P_{Hg} = \rho_{Hg} \cdot g \cdot h = 13600 \times 10 \times 0.76 = 103360$ pa.
3- Above point B there is mercury and vacuum so total pressure at point B is:
   $P_t = P_{Hg} + P_{Vaccum}$
   $P_B = P_{Hg} + Pc = 103360 + 0 = 103360$ pa.
4- The two point A and B are in same horizontal level and in same liquid so the at these two-point pressure is the same ($P_A = P_B$), P_A=103360 pa.
5- Point A is at surface of liquid so pressure that exerted at point A equal to atmospheric pressure ($P_A = P_{atm}$) => P_{atm} =103360 pa.

Example-2:

Consider a U tube containing a certain amount of water (figure 1).
Given: atmospheric pressure: Pat = 76 cm of mercury; Density of mercury: ρ_{Hg} = 13600 kg/m^3 and g = 10 N/kg.

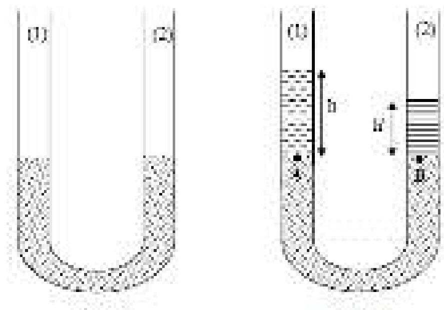


Figure 1   Figure 2

1- Calculate, in Pa the atmospheric pressure $P_{atm.}$

2- We want to determine the density ρ' of a certain liquid (L) that does not mix with water. For this reason, we pour in branch (1) of the tube an amount of oil to a height h = 20 cm and of density ρ_{oil} = 900 kg/m^3 and in branch (2) a certain amount of (L) to a height h' = 16 cm. The surfaces of separation (water-oil) and (water-liquid) are at the same horizontal plane. (Figure 2)

a) Determine, in Pa, the value of the pressure $\mathbf{P_{oil}}$ at A exerted by oil.
b) Determine, in Pa, the value of the total pressure $\mathbf{P_A}$ at A.
c) Deduce, in Pa, the value of the total pressure at B.
d) Give the expression of the total pressure $\mathbf{P_B}$ at B as a function of ρ'.
e) Deduce the value of ρ'.

solution:

given: h=76cm=0.76m, $\boldsymbol{\rho}_{Hg}$ = 13600 kg/m^3, g = 10 N/kg.

1. P_{atm}= ρ_{Hg}. g. h=13600x10x0.76=103360 pa.

2. Given: in branch-1 (oil) h = 20 cm=0.2m, ρ_{oil} = 900 kg/m^{3}in branch-2 (liquid) h' = 16 cm=0.16m, ρ'=?

a) P_{oil}= ρ_{oil}. g. h=900x10x0.2=1800pa.

b) above point A there is oil and air so total pressure at point A is:
$\mathbf{P_A}$**=** P_{oil}+ P_{atm}=103360+1800=105160pa.

c) The two point A and B are in same horizontal level and in same liquid so the at these two-point pressure is the same (P_A=P_B), P_B=105160 pa.

d) above point B there is a liquid and air so total pressure at point B is:
$\mathbf{P_B}$**=** P_{liquid}+ P_{atm}
= ρ'. g. h'+ P_{atm}
= ρ'x10x0.16+103360
=1.6 ρ' +103360

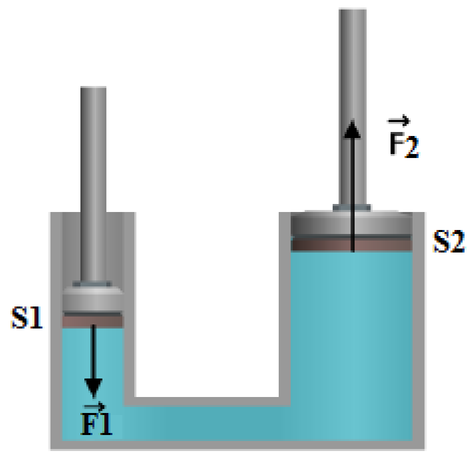
e) 105160 =1.6 ρ' +103360
ρ' = (105160-103360)/1.6
=1125 kg/m^3

5- Pascal's Theorem:

5.1- Definition

Liquid bodies transmit totally to all the points in this liquid and in **all directions and equally** any variation of pressure they undergo.

5.2- Hydraulic press:

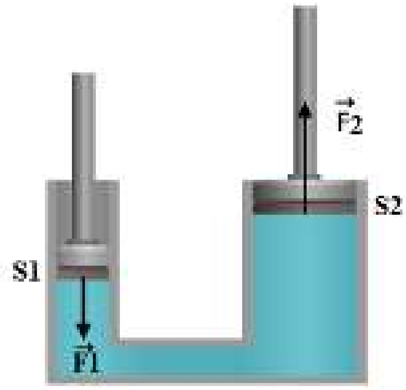


- Before pressing: ($P_A = P_{Atm}$ and $P_B = P_{Atm}$)
- After pressing: ($P_{A'} = P_{Atm} + \frac{F1}{S1}$ and $P_{B'} = P_{Atm} + \frac{F2}{S2}$).
- Variation of ΔP:
  - $\Delta P_A = P_{A'} - P_A = P_{Atm} + \frac{F1}{S1} - P_{Atm} = \frac{F1}{S1}$
  - $\Delta P_B = P_{B'} - P_B = P_{Atm} + \frac{F2}{S2} - P_{Atm} = \frac{F2}{S2}$
- **But according to pascal's theorem, Liquid must transmit ΔP from A to B, so:**

$$\Delta P_A = \Delta P_B$$
$$\frac{F1}{S1} = \frac{F2}{S2}$$

Example:

A hydraulic jack is used to lift cars. Document below shows the principle on which it works. Suppose that a downward force of magnitude F_1 = 1N acts on a piston of area $S_1 = 0.01\ m^2$. The area of the other piston is $S_2 = 0.5\ m^2$.



1- State pascal's theorem.
2- Calculate the variation of pressure **ΔP_1** transmitted through the liquid.
3- Write down the relation between magnitude of the two forces.
4- Determine the magnitude F_2 of the force acting on the other piston due to this variation.

Solution:

Given: F_1 = 1N, $S_1 = 0.01\ m^2$, $S_2 = 0.5\ m^2$.

1- Liquid bodies transmit totally to all the points in this liquid and in **all directions and equally** any variation of pressure they undergo.

2- $\Delta P_1 = \frac{F1}{S1} = \frac{1}{0.01}$ =100pa

3- **According to pascal's theorem, Liquid must transmit same variation** ΔP **from piston 1 to piston 2, so:**

$$\Delta P_1 = \Delta P_2$$
$$\frac{F1}{S1} = \frac{F2}{S2}$$

4- $\frac{F1}{S1} = \frac{F2}{S2}$ **=> $F_2 x S_1 = F_1 x S_2$ => $F_2 = \frac{F1XS2}{S1} = \frac{1x0.5}{0.01}$ =50N**